

## FIVE TURNING POINTS IN THE HISTORICAL PROGRESS OF STATISTICS—MY PERSONAL VISION

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**ABSTRACT.** Statistics has penetrated almost all branches of science and all areas of human endeavor. At the same time, statistics is not only misunderstood, misused and abused to a frightening extent, but it is also often much disliked by students in colleges and universities. This lecture discusses/covers/addresses the historical development of statistics, aiming at identifying the most important turning points that led to the present state of statistics and at answering the questions “What went wrong with statistics?” and “What to do next?”.

**1. Introduction.** There are two developments with regard to statistics. First, statistics has penetrated almost all branches of science, and secondly, statistics is misunderstood, misused and abused to a frightening extent. The first development is, of course, welcome, however, the second development is not only regrettable, but extremely dangerous and becomes even more dangerous as more fields of application are opened for statistics.

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Therefore, the questions “What went wrong with statistics?” and “What to do next?” arise, and this lecture is an attempt to answer them. The answers do not try to find, as is often done, fault with poorly educated practitioners or with inappropriate teaching of statistics in schools and universities, but look at the development of statistics from its beginnings in the 17th century to our times. The aim is to identify the turning points in its development that paved the way for the present state.

Almost 50 years ago as a student of mathematics, I attended my first course in statistics which could be described as a subject-oriented statistics (SOS) course and a happy course as recently postulated as a solution of the above outlined problems with statistics by Xiao-Li Meng [10] in a widely-noticed paper in the ASA journal *The American Statistician*. Although the course I attended was delivered by a *most qualified statistician—in terms of both teaching and research credentials*, I disliked it because some fundamental questions were not answered satisfactorily with respect to the nature of statistics, to the meaning of the concept probability and the appropriateness of statistical procedures. When consulting some textbooks, my attitude towards statistics did not become any better. Not only did I learn that in statistics the meaning of a given probability depends on several incompatible approaches, where at least one of them categorically denies the existence of randomness and, hence, that of objective probabilities, and instead tries to introduce belief into science; I also learnt how one can prove statistically both a claim A and its opposite not-A, based on the same observations. My first encounter with statistics thus resulted in serious doubts.

I later majored in quality control and began to examine the quality of statistics with respect to transparency, unambiguity and consistency. The result was not at all good! Subsequently, I tried to identify the turning points in the historical development of statistics that had led what David R. Fox [8], for example, describes in his response to Xiao-Li Meng’s above mentioned paper with the following words:

*The recent article by Meng (2009) continues a long tradition of articles in this journal [i. e., The American Statistician (TAS)] dealing with the future of statistics and statisticians. For over 25 years many of these accounts painted varying shades of the same grim picture—that our continued existence is under threat; the challenges are great; respect has been in short supply; and our future is bleak.*

In fact, the discussion about what went wrong with statistics has a long tradition and goes back to the very beginning of statistics. Fox lists the following items:

- In his Presidential address delivered on the occasion of this society's [i. e., ASA] 141st Annual Meeting in Detroit in 1981, Ralph Bradley asked 'what then is wrong with statistics and what should we do for its future?' (Bradley 1982).
- Almost 20 years later, John Nelder warned us that 'the public image of statistics is poor and may be becoming worse' and suggested that one of the biggest problems was the word 'statistics' itself. (Nelder 1999).

Meng's question "What do we do now and over the next 50 years?" shows that the related questions are still open and that it is high time to take a close look at the history of statistics in order to find the turning points.

It is widely acknowledged that the development of probability theory began with the famous correspondence between Pierre de Fermat (1607–1665) and Blaise Pascal (1623–1662), which dealt with some problems of gambling. The problems were presented to Pascal by the Chevalier de Méré (1607–1684). The subsequent exchange between Blaise Pascal and Pierre de Fermat on these questions took place in the summer of 1654. Their correspondence is considered to be the beginning of probability theory, although the word probability did not appear in these letters, and the posed problems were solved solely by combinatoric methods. However, the discussion stimulated further research about gambling and led, in fact, to the introduction of a new branch of science, based on the measure of probability by Jakob Bernoulli (1655–1705).

**2. Stochastics.** Jakob Bernoulli was an educated theologian and an autodidact in mathematics and physics. He lived in Basel in Switzerland and was appointed to the chair of mathematics at Basel university in 1687. He held the chair for 18 years until he passed away in 1705.

**2.1. Jakob Bernoulli.** Present statistics is based on probability and actually most of the difficulties in understanding and applying statistics are closely connected with this concept. The term probability<sup>1</sup> was formally defined by Jakob Bernoulli in his masterpiece *Ars conjectandi* that was published posthumously in 1713. The objective of this book was to introduce a new branch of science, named stochastics ("stochastike" is Greek and means the same as the Latin *ars*

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<sup>1</sup>Bernoulli was not the first to use the term probability (Latin: *probabilitas*), but benefitted from the so-called probabilism, which was a moral system of the Catholic Church, formally defined in 1577 by the Spanish Dominican Bartholomé de Medina (1527–1581) and mainly applied by the Jesuits.

*conjectandi* which may be translated as *Science of Prediction*<sup>2</sup>). Bernoulli was strictly against any form of divination or conjecturing, such as astrology, which was very popular at his time. He wanted to replace unscientific conjecturing by scientific predictions. Making reliable predictions is difficult, as any future development may result in different outcomes, i. e., any considered future event may or may not occur. This property of future events can easily be visualized by repeating a process that results in different outcomes. Indeed, this is a characteristic property of real-world processes and is generally ascribed to randomness. Bernoulli realized that quantification of randomness was therefore a necessary condition for developing a *Science of Prediction*.

Bernoulli lived in the 17th century when any doubt in God's omniscience and omnipotence was considered heretic and thus potentially dangerous. This may be one of the reasons for which Bernoulli did not distinguish clearly between the *contingent future* and the *fixed past and presence*<sup>3</sup> and therefore did not discuss the possibly heretic concept of randomness. He notes:

*In themselves and objectively, all things under the sun, which are, were, or will be, always have the highest certainty. This is evident concerning past and present things, since, by the very fact that they are or were, these things cannot not exist or not have existed. Nor should there be any doubt about future thing, which in like manner, even if not by necessity of some inevitable fate, nevertheless by divine foreknowledge and predetermination, cannot not be in the future. Unless, indeed, whatever will be will occur with certainty, it is not apparent how the praise of the highest Creator's omniscience and omnipotence can prevail. Others may dispute how this certainty of future occurrences may coexist with the contingency and freedom of secondary causes, we do not wish to deal with matters extraneous to our goal.*<sup>4</sup>

This quote shows why Bernoulli did not make a distinction between past and future, but only mentioned that there are dissenting opinions, without, however, being responsive to them.

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<sup>2</sup>Generally translated as *Art of conjecturing*, which, however, contradicts Bernoulli's aims to initiate scientific predictions.

<sup>3</sup>Jakob Bernoulli was a disputatious person, but knew and observed the limits set by religion and society. For example, once he accused the university board of corruption, but when the university board decided to fire him as chairholder, he withdrew his accusation.

<sup>4</sup>All translations of the *Art conjectandi* given here are taken from Edith Dudley Sylla's translation [15].

Randomness refers to the possibility for an event to occur or not. To quantify randomness, Bernoulli therefore selected an event's readiness to occur as the relevant property. Perfect readiness means that the event will occur with certainty, and no readiness at all stands for the impossibility to occur. Thus, Bernoulli quantified randomness by the measure *probability of an event*, which he defined as follows:

*probability of an event = degree of certainty of its occurrence*

and he adds that the "degree of certainty" (= probability) differs from certainty as a part differs from the whole.

Note that Bernoulli's probability is not only independent of the situation in gambling, but also of any method to measure or calculate it and, in particular, void of any mathematical concept. Thus, it applies to any event with respect to a real process. The above definition of the measure of probability is much easier to understand than most of the measures used in physics. Compare, for example, the measure of *probability* with the measure of *kilogram* which is defined as:

*kilogram of a body = resistance to change the speed of the body by an external force (inertia)*

which quantifies the mass (inertia) of a physical body and can be visualized by trying to accelerate it.

In contrast to many quantifications in physics, the rules of quantification of randomness follow immediately from its definition, i. e., one obtains:

- Unit, i. e.,  $P(E) = 1 \Leftrightarrow E$  will occur with certainty.
- Zero, i. e.,  $P(E) = 0 \Leftrightarrow E$  will not occur with certainty.
- Additivity, i. e.,  $P(E_1 \cup E_2) = P(E_1) + P(E_2) \Leftrightarrow E_1$  and  $E_2$  are mutually exclusive events.

Note that these rules of quantification, except for the upper bound, also hold for the mass of a body and many other physical measures.

Next, Bernoulli turns to the important question of how to determine (measure) the unknown value of the probability of a given event. And again, he shows surprising insight into the science of measurement which was far more advanced than the corresponding approach in metrology. Moreover, Jakob Bernoulli realized that prediction and measurement are in some sense equivalent. He notes:

*To conjecture [i. e., predict] about something is to measure its probability. Therefore we define the art of conjecture [i. e., prediction], or stochastics, as the art of measuring probabilities of things as exactly as possible . . .*

Jakob Bernoulli also realizes the difference between calculating probabilities in the context of gambling, where only the number of cases have to be determined, and the general case. Bernoulli notes:

*From this it resulted [in previous chapters on gambling] that the only thing needed for correctly forming conjectures [predictions] on many matter is to determine the number of these cases accurately and then to determine how much more easily some can happen than others. But here we come to a halt, for this can hardly ever be done. Indeed, it can hardly be done anywhere except in games of chance. The originators of these games took pains to make them equitable by arranging that the numbers of cases resulting in profit or loss be definite and known and that all cases happen equally easy. But this by no means takes place with most other effects that depend on the operation of nature or on human will.*

Finally, he turns to the question of whether it is possible to determine the true, but unknown value of the probability of a considered event.

*But what mortal, I ask, may determine, for example, the number of diseases, as if they were just as many cases, which may invade at any age the innumerable parts of the body and which imply our death? And who can determine how much more easily one disease may kill than another—the plague compared to dropsy compared to fever? Who, then, can form conjectures [predictions] on the future state of life and death on this basis? Likewise who will count the innumerable cases of the change to which the air is subject every day and on this basis conjecture [predict] a future constitution after a month, not to say after a year? Again, who has a sufficient perspective on the nature of human mind or on the wonderful structure of the body so that they would dare to determine the cases in which this or that player may win or lose in games that depend in whole or part on the shrewdness or the agility of the players? . . . In these and similar situations, since they may depend on causes that are entirely hidden and would forever mock*

*our diligence by an innumerable variety of combinations, it would clearly be mad to want to learn anything in this way.*

These words stress the *innumerable* cases which must be known in order to predict the future, or measure a probability exactly. Since this is impossible, Bernoulli, more than 300 years ago, called *mad* any attempt to do so. In contrast, modern metrology only published an official guide on the uncertainty of measurement in 1993 [9]. Moreover, all the examples given in the citation above refer to future events, supporting the interpretation of future events and thus randomness that Bernoulli had in mind when defining the measure *probability*.

Having realized that the determination of the true value is impossible, Bernoulli described how an unknown probability of a given event may nevertheless be measured.

*This empirical way of determining the number of cases by experiments is neither new nor uncommon. The author of "The Art of Thinking" [= Ars cogitandi or Port-Royal Logic.], a man of great acuteness and talent, made a similar recommendation . . . , and everyone consistently does the same thing in daily practice. Neither should it escape anyone to judge in this way concerning some future event it would not suffice to take one or another experiment, but a great abundance of experiments would be required, given that even the most foolish person, by some instinct of nature, alone and with no previous instruction (which is truly astonishing), has discovered that the more observations of this sort are made, the less danger there will be of error. But although this is naturally known to everyone, the demonstration by which it can be inferred from the principle of art is hardly known at all, and, accordingly, it is incumbent upon us to expound it here. . . . But I would consider that I had not achieved enough if I limited myself to demonstrating this one thing, of which no one is ignorant. Something else remains to think about, which perhaps no one has considered up to this point. It remains, namely, to ask whether, as the number of observations increases, so the probability increases of obtaining the true ratio between the numbers of cases in which some event can happen and not happen, such that this probability may exceed any given degree of certainty.*

The awareness of the impossibility to exactly determine the true value of a probability is expressed as follows:

*Lest, however, these things be misunderstood, it must be carefully noted that we do not wish the ratio between the numbers of cases that we have undertaken to determine by experiments to be taken precisely or as an indivisible (for, if it were, then the opposite would occur, and it would become less probably that the true ratio had been found as more observations are taken). Rather, the ratio should be defined within some range, that is, contained within two limits, which can be made as narrow as anyone might want.*

These words show that Bernoulli did not aim at developing a limit theorem, but at establishing an empirical method to determine a lower and an upper limit for an unknown probability.

In summary, it can be said that Bernoulli quantified randomness in a straightforward and generally intelligible way. By this, he was far ahead of the quantifications of physical attributes, such as mass, weight, temperature, etc., some of which only were formulated much later.

Unfortunately, Bernoulli passed away in 1705, leaving his masterpiece unfinished. Moreover, his widow and son were apprehensive of plagiarism especially by Jakob's brother Johann (1667–1748) and his nephew Nikolaus (1687–1759) and therefore kept the manuscript locked until 1713, when it was finally published by his son Nikolaus. However, this was too late! In the meantime, Pierre Rémond de Montmort (1678–1719) in France and Abraham de Moivre (1667–1754) in England had published works that did not continue Bernoulli's ideas, but returned to investigating gambling. Both of them took the term *probability* from the then unpublished *Ars conjectandi* without mentioning its aim and meaning, namely to quantify randomness and being the basis for stochastics, i. e., a *Science of Prediction*.

**3. The First Turning Point.** Not approving Bernoulli's quantification of randomness and not continuing his development of stochastics represents the first and decisive turning point in the development of a science dealing with uncertainty. It meant adopting the shell but not the content. Randomness was again reduced to gambling, and probability to a ratio of cases.

Instead of continuing Bernoulli's work by taking the next step and distinguishing between future as the realm of *contingent events* and past as the realm of *fixed facts*, further efforts focussed on theoretical investigations of games of chance, assuming uniform distribution and, thus, actually making the concept *probability* unnecessary. As a consequence, the probability of an event was not explained in principle, but only a method was given on how to calculate it, namely



as the ratio of numbers of cases. Abraham de Moivre, for example, introduced in his famous handbook for gamblers *Doctrine of Chances—or, a method for calculating the probabilities of events in play* the probability of an event as follows:

*The Probability of an Event is greater or less, according to the number of Chances by which it may happen, compared with the whole number of Chances by which it may happen or fail.*

The question “What is a probability?” was not answered in a general and unambiguous way, and this remained true until our times. Moreover, the continuation of Jakob Bernoulli’s work was explicitly refused. At the end of the Preface of the first edition of the *Doctrine of Chances* published in 1718, de Moivre mentioned Jacob Bernoulli and his project with the following words:

*Before I make an end of this Discourse, I think my self obliged to take Notice, that some years after my Specimen was printed, there came out a Tract upon the subject of Chance, being a Posthumous Work of James Bernoulli, wherein the Author has shewn a great deal of Skill and Judgement, and perfectly answered the Character and great Reputation he had so justly obtained. I wish I were capable of carrying on a Project he had begun, of applying the Doctrin of Chances to Oeconomical and Political Uses, to which I have been invited, together with Mr de Montmort, by Nicholas Bernoulli. I heartily thank that Gentleman for the good Opinion he has of me, but I willingly resign my share of that Task into better Hands, wishing that either he himself would prosecute that Design, he having formerly published some successful Essays of that kind, or that his Uncle Mr. John Bernoulli, Brother of the deceased, would be prevailed upon to bestow some of his Thoughts upon it; he being known to be perfectly well qualified in all respects for such an Undertaking.*

As a consequence, the further development of the investigation of uncertainty went off by producing more or less independent techniques with no unifying aim. It is therefore not surprising that statistics is often regarded as a methodology and not a science.

**4. Arithmetical Politics.** Almost simultaneously with Bernoulli’s more theoretical derivations, many data sets were compiled and published in Europe. The following ones are best known:

1. John Graunt's *Natural and Political Observations Made upon the Bills of Mortality* (1662): Graunt (1620–1674) developed early human statistical and census methods that later provided a framework for modern demography.
2. William Petty's *Political Arithmetick* (1690): Petty (1623–1687) proposed a method of reasoning by figures upon things relating to Government, i. e., he expressed these things in terms of numbers, weight, and measure, in order to handle them mathematically. Like his friend Graunt, Petty actually only used simple averages.
3. Johan de Witt's *Waardije van Lyf-renten naer Proportie von Los-rent* (1671) ("The Worth of Life Annuities Compared to Redemption Bonds"): de Witt (1625–1672) is regarded as the initiator of actuarial mathematics.
4. Caspar Neumann's *Reflexionen über Leben und Tod bey denen in Breslau Geborenen und Gestorbenen* (1693) "Reflections about life and death of those born and died in Breslau" had some relations to Edmond Halley (1656–1741), who used Neumann's data in his paper *An estimate of the degree of the Mortality of Mankind, drawn from curious Tables of Births and Funerals of the City of Breslaw; with an attempt to ascertain the price of Annuities upon Lives*.
5. Johann Peter Süßmilch's *Die göttliche Ordnung in den Verhältnissen des menschlichen Geschlechts, aus der Geburt, dem Tode und der Fortpflanzung desselben erwiesen* (1741) ("The Divine order in the circumstances of the human sex, birth, death and reproduction") is the first systematic work on vital statistics in Germany. For Süßmilch (1707–1767), the constancy of the number of the population was proof of the existence and providence of God.

These collections of health and political data and the conclusions drawn from them gained great popularity. Especially the connection with the Christian faith and their interpretation as being proof for the existence of God by showing that the totals or means remained more or less stable over time made these new activities extremely exciting. The idea that political decisions should be based on (objective) figures and not on secret agreements added to the attraction of this new scientific development, named *political arithmetic* by William Petty.

Unfortunately, the two developments (Bernoulli's stochastics and Petty's political arithmetic) did not merge to form an empirically justified new branch of science for making reliable predictions. Jakob Bernoulli tried in vain to obtain

Johan de Witt's manuscript, which was published in 1671. He planned to complete the *Ars conjectandi* by applying his theoretical results to de Witt's data. Without success, he asked Gottfried Wilhelm Leibniz (1646–1716) in several letters to send him Witt's manuscript. Leibniz pretended not to find the manuscript, and this was probably one of the reasons why Jakob Bernoulli never completed his masterpiece before passing away in 1705.

What can be gained by an unambiguous definition of probability? More than 30 years ago, I changed my statistics lectures by first introducing the concept of quantification to express real things, using the language of mathematics, and then by replacing the many interpretations of probability by Bernoulli's quantification of randomness. The effect was immediately observable. Not only did the students understand the aim and methods of statistics better, but they also looked at methods and models of physics and science more critically.

**4.1. Statistics.** With time passing, the public interest in *political arithmetic* as well as in the *theory of gambling* decreased. This was the hour of Sir John Sinclair (1754–1835) of Scotland. He compiled and published, between 1791 and 1799, a total of 21 volumes of his *Statistical Account of Scotland*. With this work, the term *statistics* was introduced. The reasons for changing the name from *political arithmetic* to *statistics* were explained by Sinclair himself <sup>5</sup>:

*Many people were at first surprised at my using the words “statistical” and “statistics”, as it was supposed that some term in our own language might have expressed the same meaning. But in the course of a very extensive tour through the northern parts of Europe, which I happened to take in 1786, I found that in Germany they were engaged in a species of political enquiry to which they had given the name “statistics,” and though I apply a different meaning to that word—for by “statistical” is meant in Germany an inquiry for the purposes of ascertaining the political strength of a country or questions respecting matters of state—whereas the idea I annex to the term is an inquiry into the state of a country, for the purpose of ascertaining the quantum of happiness enjoyed by its inhabitants, and the means of its future improvement; but as I thought that a new word might attract more public attention, I resolved on adopting it, and I hope it is now completely naturalised and incorporated with our language.*

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<sup>5</sup>The given citation can be found on pages xiii and xiv of the 20th volume of the *Statistical Account* [14].

Sinclair's importance for the development of statistics is appreciated in the introduction of the first issue of the *Journal of the Statistical Society of London*<sup>6</sup> by the following words:

*The History of Statistics in this country will occupy but a short space. Until within a very few years, England possessed few works of much authority embracing all the various branches of the Science. Among the few valuable labours of this kind may be mentioned Sir John Sinclair's "Statistical Account of Scotland", . . . Many other publications in particular branches of this science might be mentioned, but the first which comprehends all the details of Statistical Science was the account of Scotland . . .*

**5. The Second Turning Point.** Statistics is an artificial word without an evident meaning in contrast to Bernoulli's *stochastics*, de Moivre's *doctrine of chance* or Petty's *political arithmetic*. In the beginning, the term statistics was used in the context of collecting vital, health and social data for calculating means. Today, statistical methods are used in almost all areas of human endeavor. However, the simple question "What is statistics?" has not been answered conclusively so far, although numerous attempts have been made.

The introduction of the new name *statistics* represents the second turning point, as it does not stand for any goal or subject of investigation. Statistics may stand for everything dealing with data. The crux is that any science deals with numbers and hence could, in some sense, be looked upon as part of statistics. This is true because neither the name nor the methods clearly reveal the subject of investigation of statistics. Hence, the new name did support the lack of understanding of the new "science" and its methods.

With adopting Bernoulli's definition of probability, I also changed the name of my lectures from statistics to stochastics, i. e., "science of prediction". By this change, the students' interest was already aroused before the lectures had started, and gradually even their world view and their way of thinking changed.

**6. Social Physics.** The next step in the development of statistics is closely related to the Belgian Lambert Adolphe Jacques Quetelet (1796–1874). Studying astronomy and the theory of probability in Paris under the guidance of

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<sup>6</sup>The Statistical Society of London was founded in 1834 and in 1887 became the Royal Statistical Society (RSS).

Jean Baptiste Joseph Fourier (1768–1830) and Pierre Simon-Laplace (1749–1827) from 1824, he was also in contact with Sylvestre François Lacroix (1765–1843) and Siméon Denis Poisson (1781–1840).

**6.1. Average Man and Normal Distribution.** Statistics was extensively used in astronomy to handle measurement errors with the method of least squares. The astronomer Quetelet, probably influenced by Laplace, did not try to introduce uncertainty into physics, but on the contrary made an attempt to extend physics to the social area. He developed something he called *social physics*, and described it in his masterpiece *Sur l'homme et le développement de ses facultés, ou Essai de physique sociale* (two volumes, 1835). While Jakob Bernoulli had aimed at extending the theory of gambling to the social area (civil, moral and economic matters) by analyzing the variability of processes, Quetelet's approach denied randomness and is an attempt to reduce variability by looking for an appropriate quantity that exhibits less variation. Trained in error analysis, he immediately arrived at the mean or average. Consequently, he introduced the concept of the *average man* (l'homme moyen) who is characterized by the mean values of measured variables that follow a normal distribution. The law of error (normal distribution) seemed to him to be not only useful for astronomy but also for human beings and social phenomenon. Quetelet thought that the average physical and intellectual characteristics of the population could be measured; therefore, normal and abnormal behavior could be defined. In addition, Quetelet claimed that the variation around the mean occurred not randomly, but in a steadfast order that gave shape to a normal distribution just as in error theory. Throughout his work, Quetelet held to the notion that there was no such thing as a chance event. All phenomena were *caused* and related. For Quetelet, the regularities he observed in social laws were comparable to the laws in the natural sciences. For him, viewing large groups of people was similar to viewing physical facts, and accordingly *social physics would present laws quite as admirable as the mechanics of inanimate objects*.

Quetelet's importance to the development of statistics becomes obvious by his activities. He was a co-founder of the English *Statistical Society of London* and in 1853 organized the First International Conference on Statistics. He helped founding several national statistical organizations (among them the *American Statistical Society*) and became decisive for the development and organization of worldwide statistics in the 19th century.

**7. The Third Turning Point.** Quetelet denied the existence of randomness and insisted in determinism and causality. In order to be able to copy

the methods of physics, not the observed variability was the target, but, following physical practice, the mean that showed only relatively little variability. He also established the normal distribution as the standard model for the always existing variability, making any special research with respect to variability unnecessary.

Each of these features can still be found in nowadays statistics. The theoretical concept of the *expectation* has replaced the mean and has acquired central importance not only in statistics; the normal distribution is still used as the standard model for variability, and the widespread insistence on determinism makes a meaningful interpretation of the concept of probability almost impossible.

**8. Kolmogorov's Probability Theory.** About 50 years after Sinclair's coup, in 1838, the American Statistical Society (ASA) was founded (renamed American Statistical Association in 1939), aiming at collecting, preserving, and diffusing statistical information in the different departments of human knowledge. From its very beginning, the ASA had close affiliation with the statistical work of the U.S. Government, particularly the U.S. Census Bureau. In those days everything focused on calculations and interpretations of means, while the concept of probability played no significant role.

This has changed. Nowadays, probability and sophisticated mathematical models characterize statistics and hence the American Statistical Association. However, the simple questions "What is the meaning of the term probability?" or "What is statistics?" are still not answered. Instead different interpretations of the term *probability* prevent a meaningful clarification of statistics.

**8.1. Foundations of the Theory of Probability.** In 1933, the monograph *Grundbegriffe der Wahrscheinlichkeitstheorie (Foundations of the Theory of Probability)* written by the Russian mathematician Andrej N. Kolmogorov (1903–1987) appeared in the German mathematical journal *Ergebnisse der Mathematik*. Kolmogorov notes in the preface:

*The purpose of this monograph is to give an axiomatic foundation for the theory of probability.*

The "axiomatic foundation" did not at all aim at clarifying the meaning of the term "probability" in science, but to establish a new branch of mathematics *in exactly the same way as Geometry and Algebra*. In order to reach this aim, Kolmogorov developed this new branch of mathematics completely independently of any real world meaning. He notes:

*... the concept of a field of probabilities is defined as a system of sets which satisfy certain conditions. What the elements of this set represent is of no importance in the purely mathematical development of the theory of probability.*

In other words, Kolmogorov introduced the term *probability* detached from any real world meaning. He stressed this fact by noting:

*Every axiomatic (abstract) theory admits, as is well known, of an unlimited number of concrete [i. e., real-world] interpretations besides those from which it was derived.*

Thus, Kolmogorov laid the basis for different interpretations of the term probability and simultaneously established the possibility to get on without any interpretation, by secluding oneself to the mathematical rules as stated in the "Theory of Probability".

In fact, Kolmogorov completed a development that had already started with Blaise Pascal, namely to consider probability theory as a special branch of mathematics. The first (official) attempt in this direction was made by the English mathematician Isaac Todhunter (1820–1884) in 1865 with his book entitled "A History of the Mathematical Theory of Probability from the Time of Pascal to that of Laplace". With Kolmogorov's mathematical axiomatization of probability theory, everything seemed to be decided and the further development of probability theory became a purely mathematical one with the focus on limit theorems. From the beginning, this development aroused criticism. In 1876, for example, the English logician John Venn wrote in the preface of his essay "The Logic of Chance" the following:

*Probability has been very much abandoned to mathematicians, who as mathematicians have generally been unwilling to treat it thoroughly. They have worked out its results, it is true, with wonderful acuteness, and the greatest ingenuity has been shown in solving various problems that arose, and deducing subordinate rules. And this was all that they could in fairness be expected to do. Any subject which has been discussed by such men as Laplace and Poisson, and on which they have exhausted all their power of analysis, could not fail to be profoundly treated, so far as it fell within their province. But from this province the real principles of science have generally been excluded, or so meagrely discussed that they had better have been omitted altogether.*

*Treating the subject as mathematicians such writer have naturally taken it up at the point where mathematics would best come in play, and that of course has not been at the foundation. In the work of most writers upon the subject we should search in vain for anything like a critical discussion of the fundamental principles upon which its rules rest, the class of enquiries to which it is most properly applicable, or the relation it bears to Logic and the general rules of inductive evidence.*

This development had only been interrupted by Jakob Berboulli's stochastics, who considered it as part of science and not part of mathematics. When Jakob Bernoulli succeeded in deriving a procedure to measure the unknown value of a probability, he noted in his diary called *Meditationes*:

*I judge this result as more valuable as if I would have succeeded in finding the quadrature of the circle; because even if the latter would be completely found it would be rather useless.*

**8.2. Interpretations.** The elaboration of more and more various interpretations of the term *probability* was an immediate consequence of Kolmogorov's work. The online Stanford Encyclopedia of Philosophy describes the actual state with respect to the term probability as follows:

*Probability is virtually ubiquitous. It plays a role in almost all the sciences. It underpins much of the social sciences—witness the prevalent use of statistical testing, confidence intervals, regression methods, and so on. It finds its way, moreover, into much of philosophy. In epistemology, the philosophy of mind, and cognitive science, we see states of opinion being modeled by subjective probability functions, and learning being modeled by the updating of such functions. Since probability theory is central to decision theory and game theory, it has ramifications for ethics and political philosophy. It figures prominently in such staples of metaphysics as causation and laws of nature. It appears again in the philosophy of science in the analysis of confirmation of theories, scientific explanation, and in the philosophy of specific scientific theories, such as quantum mechanics, statistical mechanics, and genetics. It can even take center stage in the philosophy of logic, the philosophy of language, and the philosophy of religion. Thus, problems in the foundations of probability bear at least indirectly, and sometimes directly, upon central scientific, social scientific,*



and philosophical concerns. The interpretation of probability is one of the most important such foundational problems.

What strikes here is in particular the last sentence. An interpretation of probability assumes that there is something like a probability defined by some rules of calculation and it also assumes that the meaning of the calculated probability values is unclear since otherwise there would be no need to interpret them. If we look at physical quantities then the reverse process is generally used for their introduction. First the property (like inertia, weight, temperature, etc.) of an object (e. g., a physical body) is identified and named. Subsequently it is quantified and some measurement procedures are developed to determine the actual value. Obviously, proceeding like this makes an interpretation in retrospect unnecessary. The reason that this established process has not been adopted with respect to the measure of *probability* is the first turning point, when Bernoulli's approach was discarded.

As long as probability calculus was restricted to solve gambling problems, the interpretation seemed clear. However, when applied to problems of other areas especially in conjunction with the denial of the existence of randomness, the need of an interpretation of the obtained results became urgent.

As a consequence, during the 20th century statisticians began to invent an ever increasing number of different interpretations of probability. The Stanford encyclopedia of philosophy lists six main interpretations of probability (each including many different versions) together with their theoretical and practical weaknesses. However, the most serious scientific weakness is not mentioned, namely the fact that statistics is based on the concept of probability, but that neither the corresponding object nor the relevant property is stated. Instead of a clear definition, there are the following vague and not at all compatible interpretations:

- (1) Classical Probability (e. g., Pierre-Simon Laplace, 1749–1827)
- (2) Logical Probability (e. g., Rudolf Carnap, 1891–1970)
- (3) Subjective Probability (e. g., Bruno de Finetti, 1906–1985)
- (4) Frequency Interpretations (e. g., Richard von Mises, 1883–1953)
- (5) Propensity Interpretations (e. g., Karl Popper, 1902–1994)
- (6) Best-System Interpretations (e. g., David Lewis, 1941–2001)

The consequences of the ambiguity of the term probability can be seen from the following. The astrophysicist Peter Coles looks in his paper *Statistical*

*Cosmology in Retrospect* at the past, present and future association of astronomy and cosmology with statistical theory. He notes about the different interpretations of probability:

*Roughly speaking there are two competing views. Probably the most common is the frequentist interpretation, which considers a large ensemble of repeated trials. The probability of an event or outcome is then given by the fraction of times that particular outcome arises in the ensemble. In other words, probability is identified with some kind of proportion.*

*The alternative ‘Bayesian’ view does not concern itself with ensembles and frequencies of events, but is instead a kind of generalization of the rules of logic. While ordinary logic, Boolean algebra, relates to propositions that are either true (with a value of 1) or false (value 0), Bayesian probability represents the intermediate state in which one does not have sufficient information to determine truth or falsehood with certainty.*

*Notice that Bayesian probability is applied to logical propositions, rather than events or outcomes, and it represents the degree of reasonable belief one can attach. Moreover, all probabilities in this case are dependent on the assumption of a particular model.*

The object of probabilities is, within the frequentist approach, a future event with the property of being a relative frequency (of a sequence of corresponding experiments), while it is a proposition, within the Bayesian approach, with the property of being the degree of reasonable belief in it. A given relative frequency constitutes one of many possible results of randomness and, therefore, is not appropriate to describe a property of a future event, while belief is not a property of the proposition, but the observing individual. In any case, the two interpretations represent two completely different things and should therefore not be confused by using the same terminology in a scientific context.

Imagine that there would be several incompatible interpretations of the measure used for quantifying inertia. This would make a scientific handling of matter impossible. Another consequence of the different interpretations of probability is the development of an ever increasing number of different measures of uncertainty, which increases the existing confusion about predictions, conjectures and measurements.

**9. The Fourth Turning Point.** Kolmogorov's "Foundation of the Theory of Probability" constitutes the fourth turning point. It finalized the development of probability theory as a special branch of mathematics and by this cut the already loose relation between the theory of probability and reality with the following consequences:

- Models and procedures are derived since then according to mathematical criteria (for instance mathematical feasibility or as limits) without appropriate consideration of reality and the need of applications.
- The mathematization requires explicitly an interpretation of the term probability when applied, which led to many inconsistent interpretations.
- The ambiguity of statistics and the term probability led to fragmentation of statistics into many separate branches according to the different fields of application and their needs.

Each interpretation stands for a different situation, i. e., quantifies different properties of different objects. But each interpretation uses the same vocabulary, which is completely misleading, and this fundamental weakness cannot be overcome even by excellent teaching. Students and practitioners are confronted with the necessity to visualize probabilities as relative frequencies, or as degrees of belief, or as ratios of cases, etc. etc. depending on preference or interests. Statisticians themselves are often—according to my experience—more case-hardened and refuse categorically to think about or discuss the fundamental question "What is probability?"

Consequently the findings of statistical studies cannot be assessed straightforward, but are subject to often mutually contradictory interpretations. Thus, one can state that the fourth turning point drove the final nail into the coffin of statistics as part of science. This development had however started already in the early 18th century, when all the then authorities refused to take up Jakob Bernoulli's work and fell back to gambling. Maybe because of economic reasons as books on gambling just sold better than books on a science of uncertainty.

**10. The American Statistical Association (ASA).** ASA has had its parts with respect to the last three turning points, but is innocent for the first and decisive one.

During recent years ASA has made laudable attempts to disclose the weaknesses of statistics and find a way to solve the related problems, by publishing

papers in its journal *The American Statistician* (TAS) dealing with the future of statistics. However, as described in more detail below, the reasons for the fact that “the public image of statistics is poor and may be becoming worse”, as John Nelder formulated in 1999, are not searched in the foundations of statistics, but only in inadequate communicating and teaching statistics. Thus, ASA looks at and tries to fight the symptoms rather than the sources of the problem.

We will mention here only two of this kind of papers. The first one is Emery N. Brown and Robert E. Kass’s paper *What is Statistics?* [2]. The second one is Xiao-Li Meng’s already mentioned follow-up paper *Desired and Feared—What Do We Do Now and Over the Next 50 Years?* [10] and its Discussion [11].

**10.1. “What is Statistics?”** The paper deals with weaknesses in statistical training and teaching programs, and the authors demand rightly “Statistics is a wonderful field, but the way in which statisticians view it must evolve.” Moreover they correctly realize that “The essential component that characterizes the discipline is the introduction of probability to describe variation in order to provide a good solution to a problem involving the reduction of data for a specified purpose.” However the meaning of the concept *probability* is only vaguely indicated and not defined explicitly.

Instead of giving a clear answer to the self posed question, the authors put forward “statistical thinking” as the decisive goal of training:

*Thus, we suggest that the primary goal of statistical training at all levels should be to help students develop statistical thinking. What exactly do we mean by this? Different statisticians would use somewhat different words to describe what defines the essential elements of our discipline’s approach, but we believe there is general consensus about the substance, which can be stated quite concisely. Statistical thinking uses probabilistic descriptions of variability in (1) inductive reasoning and (2) analysis of procedures for data collection, prediction, and scientific inference.*

These words present no simple and easily understandable answer to the simple question “What is statistical thinking?”. In fact, the authors take refuge in indirect statements about statistical thinking like the following:

*We understand statistical thinking to be based on these two roles for probabilistic reasoning. This allows us to elaborate our definition of statistical thinking by stating that it involves two principles:*

- (1) *Statistical models of regularity and variability in data may be used to express knowledge and uncertainty about a signal in the presence of noise, via inductive reasoning.*
- (2) *Statistical methods may be analyzed to determine how well they are likely to perform.*

If I understand the paper correctly, then Brown and Kass say very rightly that *statistics necessitates statistical thinking*, which is based on the concept of *probability* where probability is used to describe variability. However, they allow for various interpretations of probability and, therefore, the meaning of describing variability remains unclear.

### 10.2. “What Do We Do Now and Over the Next 50 Years”

Like Brown and Kass, Xiao-Li Meng complains about inadequate teaching of statistics and calls for teachers who have

- (1) *extensive statistical knowledge;*
- (2) *deep understanding of statistical foundations;*
- (3) *substantial experience in statistical practice;*
- (4) *great communication skills; and*
- (5) *profound pedagogical passion.*

As shown previously, understanding statistical foundations is hardly possible, because the fundamental concept probability is not defined unambiguously. Therefore, the foundations are understood and taught differently and the same holds for “statistical knowledge”. Also “experience in statistical practice” may be counterproductive for good teaching. In many cases the statistical practice means to apply routinely statistical methods to arrive at specific answers, where the methods are often selected retrospectively in order to get the desired result. Moreover, as described above, statistical results in general give reason to differing interpretations, which again undermines understanding.

Meng describes the role of statistics as follows:

*This should be our profession’s deepest fear: we could screw up big time because it is no longer just about helping others clean up their backyards, but rather about preparing whole generations of future scientists and policy makers.*

Accordingly, Meng assumes correctly that statistics represent some essential knowledge and skills which should be taught to any scientist and policy maker. It follows that something is missing in the education of scientists and policy makers. Unfortunately, Meng fails to specify these missing “essential knowledge and skills”. Again statistical thinking seems to be one of the main required characteristics in teaching and applying statistics. Meng notes:

*To foster more statistical thinking and to effectively prevent fragmentation, we must continuously deepen our foundation as we expand our horizon. By “deepen our foundation” I mean to engage ourselves, and encourage others to do the same, in deep statistical thinking whenever possible, and not to be contented only with the methods or results we produce.*

Just as in the case of Brown and Kass, it is difficult to understand the meaning in this statement. It says that to “foster statistical thinking” one should engage oneself in “deep statistical thinking”. The addition “whenever possible” increases the lack of understanding. But then Meng again mentions the responsibility of statistics to general science. He claims:

*We statisticians, as a police of science (a label some dislike but I am proud of; see the next section), have the fundamental duty of helping others to engage in statistical thinking as a necessary step of scientific inquiry and evidence-based policy formulation.*

If statistical thinking is a “necessary step of scientific inquiry and evidence-based policy formulation”, then not only statisticians should think statistically but all scientists should be taught and educated in statistical thinking. However, this necessitates a clear description of statistical thinking on the one hand and abandoning determinism on the other.

Meng describes the faults made in science by not being able to think statistically:

*Too many false discoveries, misleading information, and misguided policies are direct consequences of mistreating, misunderstanding, and misanalyzing quantitative evidence. I am not referring to those deliberate efforts to mislead, such as infomercial statistics or unethical behavior (e. g., a highly cited author from another field told me, to my face, that he avoids precise model descriptions so readers can never be sure what he did and hence be able to challenge him). I am referring*

*to honest mistakes made by scientists and policy makers, mistakes that could easily be avoided or caught if they themselves had been “instilled” with an appropriate amount of statistical thinking and caution. I came to this realization after having worked with astronomers, engineers, geophysicists, psychiatrists, and social scientists.*

*In other word, in almost any branch of science unconsciously or consciously mistakes are made using statistics. To avoid or prevent these mistakes Meng demands better educated statisticians who could act as a police of science. This extraordinary position within science is acknowledged by Meng with the following words:*

*My response is that our professional call, and ability to prevent others from using quantitative evidence erroneously or inappropriately, is precisely what makes statistics, as a discipline, unique, wanted, and increasingly so.*

*Meng also describe the consequences of the addressed deficiencies:*

*The grossly improper assessment of variance and correlation, either out of ignorance or greed, has brought down the (financial) world. And now that the world is down around us, our professional duty compels us to do our absolute best to educate our future trainers and trainees, and through them the general scientific community and public, about what statistics can and cannot do and why it is as essential to modern civilization as an election is to a democratic society.*

*What I do not understand is that Meng does not mention the fact that especially in the science of economics and finance, statistics plays a major role and many of the wrong decisions were based on the results of sophisticated statistical methods and models developed and applied by renowned statisticians and rewarded with Nobel Prizes. Looking at the many predictions made by statisticians in the realm of economy, which never occur, but are nevertheless the basis of far-reaching decisions, I have often wished that there would be no statistics at all!*

*When I responded to Meng’s paper by praising his idea of a police of science and expressing similar ideas as in this lecture, Meng answered in a way which reminded me of de Moivre’s reaction on the invitation to continue Jakob Bernoulli’s work:*

Von Collani labeled my article as a “milestone in the development of science,” and credited me as a revolutionary in policing science. While flattered, I must confess that I am at least a mile away in seeing the pictures von Collani is painting, and I am not sure if I would make a half turn or full turn in my grave if someone puts “Chief of Science Police” on my tombstone. Von Collani apparently is questioning the entirety of modern science and statistics and wants to replace everything by “stochastic thinking,” a discussion topic that is the furthest from my original piece, certainly beyond my reflection antenna.

The mentioned “stochastic thinking” results in a very natural way from the introduction of stochastics in the sense of Jakob Bernoulli. Scientific thinking is based on certainty and the results are given by definite points, i. e., scientific thinking means to think in points. Statistical thinking means to incorporate uncertainty generated by randomness (frequency interpretation) or belief (Bayesian interpretation) and is represented by a definite probability distribution, i. e., statistical thinking means to think in given probability distributions. Stochastic thinking not only incorporates uncertainty generated by randomness but also uncertainty generated by human ignorance and is represented in sets of probability distributions, i. e., stochastic thinking means to think in sets of probability distributions. The students learn that variability is a most significant property of real processes and when they have applied the corresponding models and methods to processes, e. g., in physics, and experience that their predictions actually occur, they start on their own to think stochastically, i. e., think in sets and abandon thinking deterministically.

**10.3. ASA’s Definition of Statistics.** In 2012, ASA adopted the following definition of “Statistics” which was given by Marie Davidian and Thomas A. Louis in the journal *Science* [6]:

*Statistics is the science of learning from data, and of measuring, controlling, and communicating uncertainty; and it thereby provides the navigation essential for controlling the course of scientific and societal advances.*

At first glance, this definition looks good, but actually it is not very precise since there is probably no science that does not learn from data, while the science of measuring is called metrology and also pretends to measure, control and communicate uncertainty.



The definition picks up Xiao-Li Meng's idea of statistics being a police of science that should control scientific advances. The question arises why is it necessary to control science to prevent wrong scientific developments. The reason is obvious: science does not consider uncertainty and, therefore, wrong conclusions are predetermined. However, to fill this deplorable and dangerous situation by statistics, it would be necessary that uncertainty and hence also probability be defined in an unambiguous way. Unfortunately, this necessary condition is not met by statistics. Consequently, statistics cannot control "the course of scientific and societal advances".

**11. A Fifth Turning Point.** In philosophy, in science and in statistics the term randomness has been discussed for many centuries. One of the reasons why the discussion does not end is the fact that people still believe in determinism and strict causality and deny anything like randomness. None the less, randomness is considered in some special fields of science (even in physics!), but with many different interpretations which makes it impossible to clearly specify its meaning.

On the contrary, Bernoulli identified, at least implicitly, the variability of the outcomes of repetitions of a process as the result of randomness and described it by the degree of certitude of the occurrence of the different events which he named probability. Unfortunately, the quantitative models of science generally neglect this characteristic property of reality. Since the processes considered in classical physics exhibit only a small variability, it was nevertheless possible to arrive at useful results. However, when processes like the radioactive decay of elements with a large variability were investigated, it became necessary to admit something like randomness also in physics. The situation is completely different for medical, social or economic processes. Without considering the inherent variability, i. e., the property of randomness, quantitative models are more or less useless and must almost necessarily lead to wrong decisions.

In order to make statistics a science, it is therefore necessary to go back to Jakob Bernoulli's work and continue it by the following steps:

- (1) Quantify randomness of future events by the measure of probability as proposed by Jakob Bernoulli.
- (2) Introduce a different terminology for quantifying different concepts (like for instance "belief" in the so-called Bayesian approach).
- (3) Model uncertainty about future developments by considering the two sources

of uncertainty, namely randomness with respect to the future and ignorance with respect to the past[4].

- (4) Clarify the subject and aims of statistics by stating that the subject is uncertainty about future developments, while the goal is twofold:
  - (a) to predict reliably the future development (forecasting), and
  - (b) to reduce reliably ignorance about the past (measuring).
- (5) Enforce that probability, the unit of randomness, is included in the International System of Units (SI) as an additional base unit.
- (6) Develop appropriate quantitative models describing the variability and allow the derivation of reliable prediction and measurement procedures.

The feasibility of most of these proposals has already been successfully checked in the course of my own teaching and research on stochastics. As to teaching, many students who have to pass the final examinations in statistics at Würzburg University prefer to be examined by me in stochastics, although I am retired and do not offer the course anymore. Instead the students learn stochastics by an E-learning program that was developed by Xiaomin Zhai[17] and implemented and offered by the company Stochastikon GmbH<sup>7</sup>.

**11.1. ASA and Ethical Reponsibility.** There is probably no institution better placed to usher a new era of statistics by triggering the fifth turning point than the American Statistical Association which is the world's largest community of statisticians and calls itself "Big Tent for Statistics."

ASA has promulgated an "Ethical Guidelines for Statistical Practice" stating correctly that "The professional performance of statistical analyses is essential to many aspects of society."

Taking this statement seriously would mean to accept responsibility for statistics on the one hand and society on the other. The 20th century and the first 14 years of the 21st century have shown society's inability of cope with risks in all fields of human endeavor. The consequences are wars (from World War I to the Iraq war), terror (e. g., September 11), crises (e. g., the current bank crisis), disasters (e. g., Fukushima), the continuing environmental and social downfalls, etc., etc. Science including statistics has proven in many ways that it is not appropriate as a tool for realizing existing risks and provide adequate solutions. The main reason for this inability is the fact that uncertainty is not part of

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<sup>7</sup>[www.stochastikon.com](http://www.stochastikon.com).

science and that statistics although dealing with uncertainty has omitted defining it in a reasonable and unambiguous way. Thus, each scientific expert assessment about a possible risk is neutralized by another scientific expert opinion proving the opposite opinion.

The ability to learn is closely linked to the ability to derive mathematical models that image the essential properties of real world. Randomness which is experienced by the fact that no replication of a process leads to the same result, is a characteristic and maybe the most decisive property of reality. Unfortunately, it has not been included appropriately into science and scientific models. The above described fifth turning point shows how this deplorable state could be changed.

Each statistician can contribute to the proposed transformation by doing the following steps:

- Introducing randomness not as an philosophical concept, but as a real-world property which is experienced every day by everybody.
- Using the term probability only in the sense of Jakob Bernoulli, i. e., as quantification of randomness.
- Clearly stating the subject and aim of statistics.
- Permanently demanding that probability be included into the SI as an additional base unit.

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