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ERGODIC RATES OF THE FAST RAYLEIGH FADING RELAY CHANNEL FOR DIFFERENT COOPERATIVE STRATEGIES

Maryan Rizinski, Venceslav Kafedziski

Abstract. This paper investigates the ergodic rates of three cooperative strategies for the fast Rayleigh fading relay channel, namely, decode-and-forward (DF), compress-and-forward (CF) and amplify-and-forward (AF). The transmitter-relay, relay-receiver and transmitter-receiver links experience independent fast Rayleigh fading. Our analysis considers both full-duplex and half-duplex modes of operation of the relay channel, assuming that perfect transmit channel distribution information (CDI) is available at the transmitter and the relay. The ergodic rates are compared under different channel conditions to the corresponding upper capacity bounds of the relay channel and to the rate of direct transmission in order to investigate when it is beneficial to use a certain cooperative strategy. Cooperative transmission makes the ergodic rates approach or even become identical to the upper capacity bounds if some conditions are satisfied. The results show that DF and CF outperform AF as they are more complex strategies, while AF itself achieves higher rates than direct transmission in most channel scenarios, thus trading performance for implementation simplicity.

ACM Computing Classification System (1998): E.4, C.2.1.

Key words: Gaussian relay channel, fast Rayleigh fading, channel capacity, ergodic rates, cooperative strategies, amplify-and-forward, decode-and-forward, compress-and-forward, wireless networks.

1. Introduction. The physical layer is no longer viewed only in its traditional role – merely as a pipe for reliable information transfer between two network points. To improve performance and achieve faster and more reliable services, wireless communication systems employ neighboring users that cooperatively transmit information. In this user-cooperation paradigm the physical layer is not just a point-to-point link, but the network itself. The fundamental building block in user-cooperation is a three-terminal network, known as the relay channel. It consists of a transmitter, a receiver and a relay that aids the communication between the transmitter and the receiver.

The problem of analyzing relay channels is not new. A vast literature has been published on this subject since the introduction of the Gaussian relay channel in 1971 by van der Meulen [1]. What has remained unknown is the optimal information processing strategy at the relay; hence the relay channel capacity has still not been determined in the general case. The way information is processed at the relay node defines a particular cooperative strategy. Each strategy gives an achievable rate that is upper-bounded by the relay channel capacity.

There are several cooperative strategies known in the literature of which three are most prominent. Two of them, namely decode-and-forward (DF) and compress-and-forward (CF), were introduced for the first time in the original paper by Cover and El Gamal [2]. In DF the relay node decodes and re-encodes information before sending it to the receiver, while CF utilizes compression at the relay. Amplify-and-forward (AF) is the third strategy treated in this paper. It is the simplest of all cooperative strategies – the relay only forwards to the receiver an amplified version of the signal received from the transmitter. AF was proposed in [3] for half-duplex relays and in the context of low complexity protocols for cooperative diversity. The AF strategy for the full-duplex mode was treated in [4].

The aforementioned papers focus on fixed channel coefficients and do not address fast fading. Høst-Madsen and Zhang extended the work of Cover and El Gamal by presenting detailed analytical expressions of the achievable rates of the DF and CF strategies for both full-duplex and half-duplex relay modes [5]. Their work also analyzed the Gaussian relay channel with fast Rayleigh fading. It assumed perfect channel state information (CSI) and channel distribution information (CDI) under a total power constraint on the transmitter and relay power. However, rather than comparing the ergodic rates to each other, the perspective of the study was to conclude that there is a small gap between the upper and lower bounds of the ergodic capacity. In particular, the authors assumed equal variances of the transmitter-relay and relay-receiver channel coefficients. In [5] both fading cases were addressed by presenting upper capacity bounds and DF rates only.

The present paper systematizes and compares ergodic rate performance of all three strategies under different channel conditions. The AF strategy is studied based on our previous work for fixed channel coefficients [6]; it extended the results of [3] and generalized the half-duplex achievable rate for arbitrary transmitter and relay power levels (incorporating average power level limitations as described in [5] – something that was not taken into consideration in [3]). We assume CDI is available at the transmitter and the relay. This assumption addresses practical cases in which the sender cannot adapt to the fast changes that occur in the channel. The transmitter and relay have fixed power levels and the transmitter power is optimized with respect to the relay-transmit and relay-receive periods. In contrast, the transmitter and relay powers are optimized in [5] under a constraint on the total power. We analyze various fast fading scenarios. The ergodic rates are compared not only in terms of the transmitter-receiver SNR as is in [5], but also in terms of the statistics of the channel coefficients. In particular, the dependence of the ergodic rates and bounds on the variance of the transmitter-relay channel coefficient, while keeping constant the variance of the relay-receiver channel coefficient, is examined. Additionally, the relay channel is studied in a simple geometrical setting with collinear transmitter, relay and receiver nodes to investigate the impact of the relay position on the ergodic rates and bounds. The behavior of the three strategies is presented graphically for all analyzed channel scenarios. The results provide insight into how the ergodic rates perform with respect to the upper capacity bounds and direct transmission and when it is beneficial to use a certain cooperative strategy. The conclusions are more general and sometimes different compared to [5], where the assumed channel statistics correspond to a geometry where the relay is half way between the transmitter and the receiver.

This paper is organized as follows. Section 2 explains the model of the fast Rayleigh fading relay channel for different modes of transmission and describes the three cooperative strategies. Section 3 presents analytical expressions for capacity bounds and ergodic rates. Section 4 provides discussion regarding the simulation results obtained for different channel conditions. Section 5 concludes the paper.

2. Channel model. The model of the fast Rayleigh fading relay channel is given in Fig. 1. The transmitter (node 1) sends a message to the receiver (node 3). The transmission is assisted by a relay (node 2). The powers of the transmitter and the relay are P_1 and P_2 correspondingly.

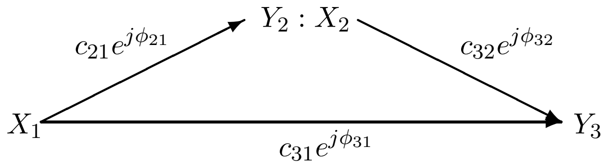

Fig. 1. The fast Rayleigh fading relay channel

The received signals at the relay and the receiver in full-duplex mode are given by

$$y_2[i] = \boldsymbol{c_{21}} x_1[i] + z_2[i] \tag{1}$$

$$y_3[i] = \boldsymbol{c_{31}} x_1[i] + \boldsymbol{c_{32}} x_2[i] + z_3[i] \tag{2}$$

where $\boldsymbol{c_{ij}} = c_{ij} e^{j\phi_{ij}}$ are the complex channel coefficients, and z_2 and z_3 are zero-mean circularly-symmetric complex Gaussian noise processes with variances N_2 and N_3 respectively. We set $N_2 = N_3 = N = 1$.

The channel in half-duplex mode is described by similar relations, except that reception at the receiver takes place in the subsequent $(i+1)$-th interval (see relation (2)).

We analyze the relay channel from the perspective of wireless communications. The communication over a wireless channel is dynamic and characterized by a set of physical phenomena that cause time-variations of the complex channel coefficients. If the channel varies significantly during the time-frame of communication, we say that the channel experiences fast fading. More specifically, we say that the channel is in a fast fading mode if a codeword spans many coherence periods (T_c). Whether there will be fast fading in the channel, depends not only on the environment, but also on the application.

We assume that the transmitter-relay, relay-receiver and transmitter-receiver links experience independent fast Rayleigh fading. In this model, the channel coefficients $\boldsymbol{c_{ij}}$ are modeled as zero-mean circularly-symmetric complex Gaussian random variables with variances $s_{ij} = \mathbb{E}[|\boldsymbol{c_{ij}}|^2]$.

We consider full-duplex and half-duplex modes of operation. The relay channel operates in full-duplex mode if the relay node can receive and transmit simultaneously. The relay channel operates in half-duplex mode if transmission and reception take place in different frequency bands or different time intervals. It is known that the half-duplex mode is the one that can be easily implemented in practical systems [5].

We also assume that transmit channel distribution information is available at the transmitter and the relay. This models many practical scenarios in which the transmitter is not able to adapt to the fast changes that occur in the channel. We consider a scenario where the channel coefficients have different variances. A simpler fading scenario where all channel coefficients have equal variance was analyzed in [7]. The CDI assumption makes the relay channel asynchronous (the transmitter and relay do not have phase information of the corresponding channel coefficients).

A specific cooperative strategy depends on the way information is processed at the relay node. The optimal information processing at the relay is unknown. Therefore, the determination of the Gaussian relay channel capacity still remains an open problem. In this paper, we consider three known cooperative strategies, namely, decode-and-forward, compress-and-forward and amplify-and-forward, whose rates also present lower bounds on the relay channel capacity.

The DF strategy is such that the relay first fully decodes the signal received from the transmitter, re-encodes it and then forwards it to the receiver. The relay might use a different codebook than the transmitter. The receiver decodes the message combining the signals that it receives from the transmitter and the relay [8].

In the CF strategy, the relay compresses the signal x_1 received from the transmitter within a certain distortion. The received signals at the relay y_2 and at the receiver $y_3^{'}$ are correlated since they are copies of the same signal x_1 obtained from two independent paths with noise and path loss. The relay uses Wyner-Ziv coding to compress y_2 treating $y_3^{'}$ as side information [9]. The compressed signal $\hat{y}_2$ is encoded in $\hat{x}_2$ which is sent to the receiver. The receiver then combines $\hat{x}_2$ and $y_3^{'}$ to decode the source message.

The simplest of all cooperative strategies is the AF strategy. As the name suggests, in this strategy the relay only amplifies the signal received from the transmitter before forwarding it to the receiver. The receiver then combines the two signals received from the transmitter and the relay. The amplification of the signal at the relay is bounded by its power constraint.

3. Expressions for capacity bounds and ergodic rates. This section presents upper bounds on the relay channel capacity and ergodic rates of the three cooperative strategies for the fast Rayleigh fading scenario described previously. C^{+} denotes an upper capacity bound and R denotes an ergodic rate of a specific cooperative strategy.

A. Full-duplex case. Applying the results in [5] on the Gaussian relay channel with CDI under fast Rayleigh fading, one may find the upper capacity bound C^+ and the ergodic rates R_{DF} and R_{CF} of the DF and CF strategies.

The upper capacity bound C^+ is

$$C^+ = \min\left(C_1^+, C_2^+\right) \tag{3}$$

where C_1^+ and C_2^+ are given by

$$C_1^+ = \mathbb{E}\left[\frac{1}{2}\log\left(1 + (c_{21}^2 + c_{31}^2)P_1\right)\right] \tag{4}$$

$$C_2^+ = \mathbb{E}\left[\frac{1}{2}\log\left(1 + c_{31}^2 P_1 + c_{32}^2 P_2\right)\right]. \tag{5}$$

The ergodic rate R_{DF} of the decode-and-forward strategy is

$$R_{DF} = \min\left(R_1, R_2\right) \tag{6}$$

where R_1 and R_2 are given by

$$R_1 = \mathbb{E}\left[\frac{1}{2}\log\left(1 + \max(c_{21}^2, c_{31}^2)P_1\right)\right] \tag{7}$$

$$R_2 = \mathbb{E}\left[\frac{1}{2}\log\left(1 + c_{31}^2 P_1 + c_{32}^2 P_2\right)\right]. \tag{8}$$

The ergodic rate R_{CF} of the compress-and-forward strategy is given by

$$R_{CF} = \mathbb{E}\left[\frac{1}{2}\log\left(1 + c_{31}^2 P_1 + \frac{c_{21}^2 P_1}{1 + \frac{c_{31}^2 P_1 + c_{21}^2 P_1 + 1}{c_{32}^2 P_2}}\right)\right]. \tag{9}$$

As it was explained in the previous section, in the amplify-and-forward strategy the relay sends to the receiver an amplified version of the signal that it receives in the previous time interval

$$x_2[i] = a y_2[i-1] \tag{10}$$

where a is the gain factor that has to meet the constraint on the relay power

$$0 \leq |a|^2 \leq \frac{P_2}{N_2 + P_1 \mathbb{E}\left[c_{21}^2\right]}. \tag{11}$$

By combining the relations (2) and (10) we get

$$y_3[i] = h_0 x_1[i] + h_1 x_1[i-1] + w[i] \tag{12}$$

where $h_0 = \boldsymbol{c_{31}}$, $h_1 = a\boldsymbol{c_{32}}\boldsymbol{c_{21}}$ and $w[i] = a\boldsymbol{c_{32}}z_2[i-1] + z_3[i]$ is AWGN noise with variance $N_w = |a\boldsymbol{c_{32}}|^2 + 1$.

We conclude that the full-duplex mode turns the Gaussian relay channel into a unit-memory intersymbol interference channel [4]. That is in fact a time-invariant frequency selective channel, that could be transformed by Discrete Fourier Transform (DFT) into a parallel Gaussian channel in frequency domain consisting of N_c subchannels

$$\tilde{y}_n = \tilde{h}_n \tilde{x}_n + \tilde{w}_n, \quad n = 0, 1, \dots, N_c - 1 \tag{13}$$

where $\tilde{\mathbf{h}} = [\tilde{h}_0, \tilde{h}_1, \dots, \tilde{h}_{N_c-1}]$ is defined as an N_c-point DFT of the zero-padded channel vector of length N_c $\mathbf{h} = [h_0, h_1, 0, \dots, 0]$, multiplied by $\sqrt{N_c}$ [10]. $\tilde{\mathbf{y}}$, $\tilde{\mathbf{x}}$ and $\tilde{\mathbf{w}}$ are DFT vectors of the output, input and noise vectors correspondingly. The noise obtained after DFT preserves the same variance N_w as before and is independent in different subchannels.

When the transmitter does not have knowledge of the instantaneous channel coefficients $|\tilde{h}_n|$, but only of their statistics, it allocates equal power in every subchannel. It can be shown that the capacity of the parallel channel in this case is given by

$$C'_{N_c} = \frac{1}{N_c} \sum_{n=0}^{N_c-1} \frac{1}{2} \log \left(1 + \frac{P_1 |\tilde{h}_n|^2}{N_c N_w} \right). \tag{14}$$

The AF ergodic rate is obtained when C'_{N_c} is averaged, followed by maximization of the relay gain factor a

$$R_{AF} = \max_a \mathbb{E}[C'_{N_c}]. \tag{15}$$

The maximization of R_{AF} with respect to a is performed numerically.

B. Half-duplex case. For a given time window D in half-duplex mode, the relay receives information for a fraction of time αD (relay-receive period) and transmits information in the remaining fraction $(1-\alpha)D$ (relay-transmit period), where $0 \leq \alpha \leq 1$. We assume that the transmitter has power $P_1^{(1)}$ and $P_1^{(2)}$ in the relay-receive and relay-transmit periods respectively.

Based on the results in [5], one may find the upper capacity bounds and the ergodic rates of the DF and CF strategies.

The upper capacity bound C^+ is

$$C^+ = \min \left(C_1^+, C_2^+ \right) \tag{16}$$

where C_1^+ and C_2^+ are given by

$$(17)\quad C_1^+ = \mathbb{E}\left[\frac{\alpha}{2}\log\left(1+(c_{31}^2+c_{21}^2)P_1^{(1)}\right)+\frac{1-\alpha}{2}\log\left(1+c_{31}^2P_1^{(2)}\right)\right]$$

$$(18)\quad C_2^+ = \mathbb{E}\left[\frac{\alpha}{2}\log\left(1+c_{31}^2P_1^{(1)}\right)+\frac{1-\alpha}{2}\log\left(1+c_{31}^2P_1^{(2)}+c_{32}^2P_2\right)\right].$$

The ergodic rate R_{DF} of the decode-and-forward strategy is

$$(19)\qquad R_{DF} = \min\left(R_1^+, R_2^+\right)$$

where R_1^+ and R_2^+ are given by

$$(20)\quad R_1^+ = \mathbb{E}\left[\frac{\alpha}{2}\log\left(1+\max(c_{31}^2,c_{21}^2)P_1^{(1)}\right)+\frac{1-\alpha}{2}\log\left(1+c_{31}^2P_1^{(2)}\right)\right]$$

$$(21)\quad R_2^+ = \mathbb{E}\left[\frac{\alpha}{2}\log\left(1+c_{31}^2P_1^{(1)}\right)+\frac{1-\alpha}{2}\log\left(1+c_{31}^2P_1^{(2)}+c_{32}^2P_2\right)\right].$$

The ergodic rate R_{CF} of the compress-and-forward strategy is given by

$$(22)\quad R_{CF} = \mathbb{E}\left[\frac{\alpha}{2}\log\left(1+c_{31}^2P_1^{(1)}+\frac{c_{21}^2}{1+\sigma_w^2}P_1^{(1)}\right)+\frac{1-\alpha}{2}\log\left(1+c_{31}^2P_1^{(2)}\right)\right]$$

where σ_w^2 is the so-called “compression noise”

$$(23)\qquad \sigma_w^2 = \frac{c_{21}^2P_1^{(1)}+c_{31}^2P_1^{(1)}+1}{\left(\left(1+\frac{c_{32}^2P_2}{1+c_{31}^2P_1^{(2)}}\right)^{(1-\alpha)/\alpha}-1\right)(c_{31}^2P_1^{(1)}+1)}.$$

It can be shown that for the amplify-and-forward strategy, the half-duplex mode turns the Gaussian relay channel into a MIMO 2×2 channel [11]. The capacity of this equivalent channel is given by

$$(24)\quad C(a) = \frac{1}{4}\log\left(1+\frac{c_{31}^2P_1^{(1)}}{N}+\frac{a^2c_{32}^2c_{21}^2P_1^{(1)}+c_{31}^2P_1^{(2)}}{(a^2c_{32}^2+1)N}+\frac{c_{31}^4P_1^{(1)}P_1^{(2)}}{(a^2c_{32}^2+1)N^2}\right).$$

To be able to compare the half-duplex mode with the full-duplex mode, we have to limit the transmitter and the relay power by setting average power constraints P_1 and P_2. Since the relay transmits information only in the relay-transmit period of length $(1-\alpha)D$, it can use power $\dfrac{P_2}{1-\alpha}$ during the transmission. Similarly, the transmitter uses power $\dfrac{\kappa P_1}{\alpha}$ during the relay-receive period and power $\dfrac{(1-\kappa)P_1}{1-\alpha}$ during the relay-transmit period, where $0 \le \kappa \le 1$ so that the average power constraint is satisfied. Assuming that the parameters α, $P_1^{(1)}$, $P_1^{(2)}$ and P_2 are fixed, we can denote the corresponding capacity bound (ergodic rate) by $C_R(\alpha, P_1^{(1)}, P_1^{(2)}, P_2)$. With these assumptions, the capacity in the half-duplex case is defined as follows

$$C(P_1, P_2) = \max_{0\le\alpha\le1, 0\le\kappa\le1} C_R\left(\alpha, \frac{\kappa P_1}{\alpha}, \frac{(1-\kappa)P_1}{1-\alpha}, \frac{P_2}{1-\alpha}\right). \tag{25}$$

This relation can be applied to all three strategies and to the upper bound in the half-duplex mode (relations (16)-(24)). The optimization of α and κ is done numerically. We set $\alpha = \dfrac{1}{2}$ for the amplify-and-forward strategy because the relay-receive and relay-transmit periods of this strategy are intrinsically equal.

To calculate the ergodic amplify-and-forward rate R_{AF}, the parameters a and κ have to be chosen optimally for all realizations (for the whole fading process). In other words, R_{AF} is obtained when $C(a)$ is averaged, followed by maximization of the relay gain a and the parameter κ

$$R_{AF} = \max_{a,\kappa} \mathbb{E}[C(a)] \tag{26}$$

where a satisfies similar bounds as in the full-duplex mode

$$0 \le |a|^2 \le \frac{P_2}{N_2 + P_1^{(1)}\mathbb{E}\left[c_{21}^2\right]}. \tag{27}$$

4. Simulation results. In this section we investigate the behavior of the ergodic rates of the three cooperative strategies and their corresponding upper capacity bounds under different channel conditions. The numerical precision of the results derived from the following graphs depends on the density of points analyzed on the horizontal axis. All results are compared to the rate of direct transmission $C = \mathbb{E}\left[\frac{1}{2}\log\left(1 + c_{31}^2 SNR\right)\right]$. FD and HD stand for full-duplex and half-duplex respectively.

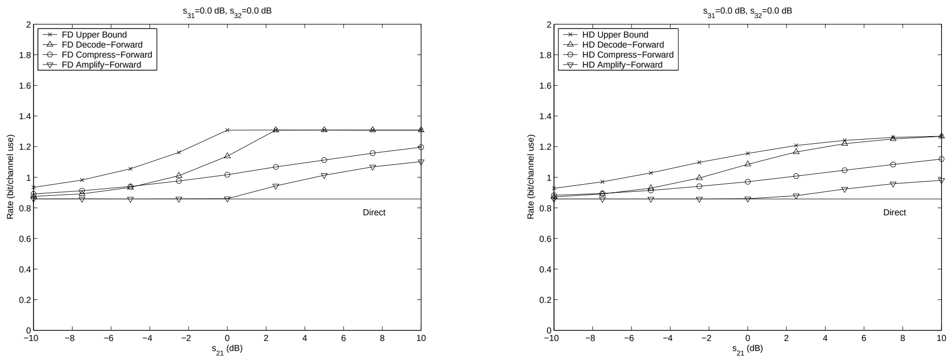

Fig. 2. Capacity bounds and ergodic rates for fast Rayleigh fading in terms of s_{21}. "Direct" is the rate of direct transmission

Fig. 2 presents capacity bounds and ergodic rates of the fast Rayleigh fading relay channel in terms of s_{21}. We assume that $s_{31} = s_{32} = 0$dB and $P_1 = P_2 = 5$dB. It can be noticed that, as s_{21} increases, the DF ergodic rates for the full-duplex and half-duplex modes approach the corresponding upper capacity bounds. If s_{21} is large enough, the upper capacity bound and the DF ergodic rate for full-duplex mode become equal to a constant value. This follows from the relations (16)-(21). When s_{21} is large enough, C_1^+ becomes larger than C_2^+, so the minimization in (16) reduces C^+ to C_2^+. On the other hand, C_2^+ does not depend on c_{21} and it results in a constant value. Similar observation can be made for the rate R_{DF}. The DF full-duplex rate and the upper full-duplex bound take equal value for $s_{21} > 2.5$dB. The DF half-duplex rate gets close to the upper half-duplex bound starting from $s_{21} = 7.5$dB. As s_{21} increases (i.e. the distance between the transmitter and the relay decreases), the DF rate increases with respect to the CF rate in both full-duplex and half-duplex regimes. The DF rate intersects the CF rate at $s_{21} = -5$dB and $s_{21} = -7.5$dB for the full-duplex and half-duplex modes respectively. When s_{21} takes small values, the upper capacity bounds for full-duplex and half-duplex modes approach each other. Also, for small s_{21} the ergodic rates of all three strategies tend to approach the rate of direct transmission. It can be noticed that the AF rates are lower than the DF and CF rates. For $s_{21} < 0$dB, both AF rates for full-duplex and half-duplex modes are identical to the rate of direct transmission.

Similar conclusions can be drawn from Fig. 3 where the capacity bounds and ergodic rates of the fast Rayleigh fading relay channel are given in terms of the SNR between the transmitter and the receiver. We consider a scenario where

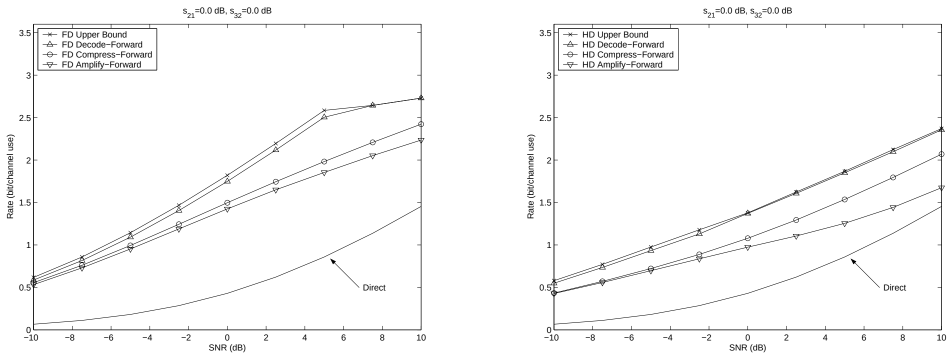

Fig. 3. Capacity bounds and ergodic rates for fast Rayleigh fading in terms of the SNR between the transmitter and the receiver

the three nodes lie on a same line such that the transmitter-receiver distance is normalized to 1. We assume $s_{31} = 1$, $s_{21} = \frac{1}{d^\gamma}$ and $s_{32} = \frac{1}{(1-d)^\gamma}$, where γ is the path loss exponent which depends on terrain and other environmental factors. We adopt the value $\gamma = 4$ which is usually used to model propagation in metropolitan areas. We also assume that $d = \frac{1}{2}$. Since $s_{31} = 1$ (0dB) and the noise variance equals 1, we have SNR $= P_1$. We assume that $P_2 =$ 5dB.

Both DF rates in the full-duplex and half-duplex modes are close to their corresponding upper capacity bounds. The DF full-duplex rate is identical to the upper full-duplex bound for SNR $>$ 7.5dB. The DF half-duplex rate is close to the upper half-duplex bound for all SNR's taken into consideration. The DF half-duplex rate is larger than the CF half-duplex rate for all SNR's. At rate $R = 1$ the SNR difference between these rates is around 3dB. On the other hand, for small SNR's, the AF half-duplex rate approaches the CF half-duplex rate. At high SNR, this difference increases. Similar observations can be made for the full-duplex mode as well. Again, it can be noticed that the AF strategy gives lower rates compared to the DF and CF strategies. However, all three strategies offer high gains with respect to direct transmission. For instance, the AF strategy in full-duplex mode offers SNR gain of around 11dB at rate $R = 1$. The AF strategy in half-duplex mode offers SNR gain of around 6dB over direct transmission at $R = 1$. Also, at $R = 1$, the DF strategy offers gain of 12dB and 10dB over direct transmission in the full-duplex and half-duplex modes respectively.

Figure 4 presents capacity bounds and ergodic rates of the fast Rayleigh fading relay channel in terms of the transmitter-relay distance. We consider the

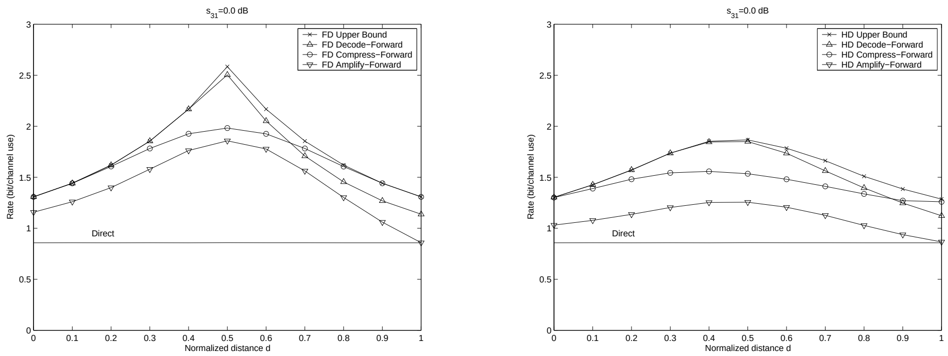

Fig. 4. Capacity bounds and ergodic rates for fast Rayleigh fading in terms of the distance d between the transmitter and the relay

same geometrical setting as in Fig. 3 and assume that $P_1 = P_2 = 5$dB. The DF full-duplex rate is identical to the upper full-duplex bound for $0 \le d \le 0.4$. The DF full-duplex rate intersects the CF full-duplex rate at $d = 0.66$. As d increases, the CF full-duplex rate exceeds the DF full-duplex rate. The CF full-duplex rate approaches the upper full-duplex bound as d approaches 1. It can be seen that the CF full-duplex rate is close to the upper full-duplex rate in the intervals $0 \le d \le 0.2$ and $0.8 \le d \le 1$. Both AF rates in the full-duplex and half-duplex regimes are lower than the corresponding DF and CF rates. As d approaches 1, both AF rates approach the rate of direct transmission. Also, it can be noticed that the DF half-duplex rate is identical to the upper half-duplex bound for $0 \le d \le 0.4$. The DF half-duplex rate intersects the CF half-duplex rate at $d = 0.87$. It is obvious that all three cooperative strategies provide higher rates than the rate of direct transmission.

5. Conclusion. We investigate the behavior of three cooperative strategies for the fast Rayleigh fading relay channel, namely, decode-and-forward, compress-and-forward and amplify-and-forward. The links between the transmitter, relay and receiver experience independent fast Rayleigh fading. We consider both full-duplex and half-duplex modes of operation of the relay channel. The channel model assumes that perfect transmit channel distribution information is available at the transmitter and the relay. The transmitter and the relay have fixed power levels. We analyze the cooperative strategies in terms of ergodic rate performance under various channel scenarios. We compare the ergodic rates to the corresponding upper capacity bounds of the relay channel as well as to the rate of direct transmission. We analyze the ergodic rates and their capacity bounds in terms of s_{21} that represents the variance of the transmitter-relay channel coefficient. With the increase of s_{21}, the DF rates approach the corresponding upper capacity bounds. At sufficiently large s_{21}, each of the DF rates becomes equal to a constant value. For small s_{21}, the ergodic rates of all three strategies tend to approach the rate of direct transmission. As s_{21} increases, the DF rates increase with respect to the CF rates. AF achieves lower rates compared to DF and CF. We also analyze the channel in terms of the SNR between the transmitter and the receiver. Both DF rates are close to their corresponding upper capacity bounds and greater than the CF rates for all SNR's taken into consideration. At low SNR, the AF rates approach the CF rates, where all rates tend to meet the performance of their upper bounds. All three strategies offer high SNR gains and perform significantly better than direct transmission. We investigate the impact of the transmitter-relay distance d as well. The collinear geometrical scenario where the three nodes lie on a same line shows again that cooperation results in higher rates compared to direct transmission. All three strategies reach their best performance when the relay is half way between the transmitter and the receiver. DF performs better than CF, and even achieves the upper bound when the relay is approaching the transmitter. CF performs better when the relay is close to the receiver. Future work will study the outage probabilities of these three cooperative strategies for the slow Rayleigh fading relay channel.

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Maryan Rizinski
ProSyst Software GmbH
Str. Vladayska 48
1606 Sofia, Bulgaria
e-mail: `m.rizinski@prosyst.com`

Venceslav Kafedziski
Faculty of Electrical Engineering
and Information Technologies
University Sts. Cyril and Methodius
Skopje, Republic of Macedonia
e-mail: `kafedzi@feit.ukim.edu.mk`

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