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ON EXTREMAL BINARY DOUBLY-EVEN SELF-DUAL CODES OF LENGTH 88*

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Abstract. In this paper we present 35 new extremal binary self-dual doubly-even codes of length 88. Their inequivalence is established by invariants. Moreover, a construction of a binary self-dual $[88, 44, 16]$ code, having an automorphism of order 21, is given.

1. Introduction. Binary self-dual codes are an interesting class of codes for several reasons. These codes include the extended $[8, 4, 4]$ Hamming code, the extended binary Golay code and the extended binary quadratic residue codes. Many of the self-dual codes are related to block designs, graphs, lattices and other combinatorial structures.

All binary self-dual codes of Type II of length up to 32 and all of Type I of length up to 34 are classified and given in [1], [2], [11], [12] and [15]. It is known that with increasing length the number of self-dual codes grows very fast. For example, there are 85 inequivalent self-dual codes of Type II of length 32 and

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at least 17000 of length 40. Therefore, the question of classifying all self-dual codes of a given length loses interest for increasing values of n.

In this work we study extremal binary doubly-even self-dual codes of length 88 having an automorphism of order 21. The greatest length of an extremal doubly-even self-dual code with minimum distance 16 is 88. The first example of such a code is given in [9, p. 633]. The next 33 codes are presented in [6]. A construction of a self-dual [88,44,16] code having an automorphism of order 5 is given in [5] and 36 new codes are listed. Here we construct 35 new binary [88,44,16] doubly-even self-dual codes. These codes and the previously known 70 codes are inequivalent.

A binary $[n, k]$ code $\mathcal{C}$ is a k-dimensional vector subspace of $\mathbb{F}_2^n$, where $\mathbb{F}_2$ is the field of two elements. The weight of a vector is the number of its nonzero coordinates. An $[n, k, d]$ code is an $[n, k]$ code with minimum weight d. A code $\mathcal{C}$ is *self-dual* if $\mathcal{C} = \mathcal{C}^\perp$ where $\mathcal{C}^\perp$ is the dual code of $\mathcal{C}$ under the standard inner product. A self-dual code $\mathcal{C}$ is *doubly-even* if all codewords of $\mathcal{C}$ have weight divisible by four, and *singly-even* if there is at least one codeword of weight $\equiv 2 \pmod 4$. Self-dual doubly-even codes exist only when n is a multiple of eight. It is known [13] that for a self-dual $[n, n/2, d]$ code:

$$d \leq 4\left[\frac{n}{24}\right] + 4, \text{ if } n \not\equiv 22 \pmod{24}, \tag{1}$$

and

$$d \leq 4\left[\frac{n}{24}\right] + 6, \text{ if } n \equiv 22 \pmod{24}. \tag{2}$$

If n is a multiple of 24, then any code reaching limit (1) must be doubly-even.

Self-dual codes which reach these bounds are called *extremal*.

The weight enumerator of a $[n, k]$ code is the polynomial $\sum_{i=1}^{n} A_i y^i$, where A_i is the number of the codewords of weight i. The weight enumerator of extremal doubly-even self-dual codes of a given length is uniquely determined [4]. Two binary codes are *equivalent* if one can be obtained from the other by a permutation of the coordinates. A permutation σ of n elements is an automorphism of a code $\mathcal{C}$ if $\mathcal{C}$ coincides with its image $\sigma(\mathcal{C})$. The set of all automorphisms of a code $\mathcal{C}$ forms the automorphism group $Aut(\mathcal{C})$ of $\mathcal{C}$.

In the next section we investigate the possible types of automorphisms of order 21 of a binary doubly-even [88,44,16] self-dual code. Further, we present a construction of a binary [88,44,16] self-dual code having an automorphism of order 21, and at last we list the new 35 codes.

2. Automorphisms of order 21 of a binary doubly-even [88, 44, 16] self-dual code. Let $\mathcal{C}$ be a [88,44,16] self-dual code having an automorphism σ of order 21. Then, σ is a permutation and, without loss of generality we may write

$$\begin{aligned}\sigma = {} & \Omega_1\Omega_2\ldots\Omega_{t_1}\ \Omega_{t_1+1}\Omega_{t_1+2}\ldots\Omega_{t_1+t_2}\\ & \Omega_{t_1+t_2+1}\Omega_{t_1+t_2+2}\ldots\Omega_{t_1+t_2+t_3}\\ & \Omega_{t_1+t_2+t_3+1}\Omega_{t_1+t_2+t_3+2}\ldots\Omega_{t_1+t_2+t_3+f},\end{aligned} \tag{3}$$

where Ω_i is a cycle of length 3 for $1 \le i \le t_1$, a cycle of length 7 for $t_1+1 \le i \le t_1+t_2$, and a cycle of length 21 for $t_1+t_2+1 \le i \le t_1+t_2+t_3$. For $t_1+t_2+t_3+1 \le i \le t_1+t_2+t_3+f$ the symbol Ω_i represents a fixed point. For short we say that σ is of type 21-$(t_1,t_2,t_3;\ f)$. From [3, Proposition 3.1] it follows that $\mathcal{C}$ has also automorphisms of type $3-(7t_3+t_1;7t_2+f)$ and type $7-(3t_3+t_2;3t_1+f)$. The possible automorphisms of order 3 and order 7 of an extremal self-dual binary code of length 88 are of types $3-(28;\ 4)$, $3-(26;\ 10)$, $3-(24;\ 16)$, $3-(22;\ 22)$, $3-(16;\ 40)$, $3-(14;\ 46)$, $7-(12;\ 4)$ and $7-(11;\ 11)$ [14, Theorem 1]. Hence, the type of the automorphism σ can be $21-(0,0,4;\ 4)$, $21-(1,3,3;\ 1)$, $21-(0,6,2;\ 4)$, $21-(2,5,2;\ 5)$, $21-(0,5,2;\ 11)$, $21-(1,2,3;\ 8)$ and $21-(3,2,3;\ 2)$.

Similar to [3] we define

$$F_\sigma(\mathcal{C}) = \{v \in \mathcal{C} \mid\ v\sigma = v\} \tag{4}$$

and

$$\begin{aligned}E_\sigma(\mathcal{C}) = \{v \in \mathcal{C} \mid\ & wt(v|\Omega_i) \equiv 0 \pmod 2,\\ & i = 1,\ldots,t_1+t_2+t_3+f\},\end{aligned} \tag{5}$$

where $v|\Omega_i$ is the restriction of v to Ω_i.

It is clear that $v \in F_\sigma(\mathcal{C})$, if and only if $v \in \mathcal{C}$ and the coordinates of v are constant on each cycle Ω_j, $j = 1,2,\ldots,t_1+t_2+t_3+f$. The map π is defined by

$$\pi:\ F_\sigma(\mathcal{C}) \to \mathbb{F}_2^{t_1+t_2+t_3+f}, \qquad \pi(v|\Omega_i) = v_j, \tag{6}$$

for some $j \in \Omega_i$, $i = 1,2,\ldots,t_1+t_2+t_3+f$.

Let σ be of type $21-(0,5,2;\ 11)$. Then, $\pi(F_\sigma(\mathcal{C}))$ is a binary self-dual [18,9] code [3, Proposition 3.2].

Theorem 1 [10, Theorem 11]. *Let $\mathcal{C}$ be a self-dual code of length $n = n_a + n_b$ over $GF(q)$. Partition the generator matrix of $\mathcal{C}$ as follows:*

$$\begin{matrix} & n_a \; n_b \\ \begin{matrix} k_a \\ k_b \\ k_d \end{matrix} & \begin{pmatrix} A & O \\ O & B \\ D & E \end{pmatrix} \end{matrix},$$

where k_a and k_b are to be chosen as large as possible. Then

i) $k_d = \text{rank } D = \text{rank } E$,

ii) $k_b = 1/2n - (n_a - k_a)$,

iii) the code generated by the rows of A and D is the dual of the code generated by the rows of A.

Therefore, the generator matrix of any binary [18, 9] self-dual code can be presented in the form:

$$\begin{matrix} & 7 \; 11 \\ \begin{matrix} k_a \\ k_b \\ k_d \end{matrix} & \begin{pmatrix} A & O \\ O & B \\ D & E \end{pmatrix} \end{matrix},$$

where $k_b = 2 + k_a$. Then, $k_b \geq 2$. If a binary self-dual [18, 9] code generates $\pi(F_\sigma(\mathcal{C}))$, then the matrix B generates $[11, k_b, d_b]$ code where $k_b \geq 2$, $d_b \geq 16$. So, the automorphism σ is not of type $21 - (0, 5, 2;\ 11)$.

In a similar way one can show that the automorphism σ is not of type $21 - (1, 2, 3;\ 8)$ either.

Therefore if an extremal binary [88, 44, 16] self-dual doubly-even code has an automorphism of order 21, then its type is $21 - (0, 0, 4;\ 4)$, $21 - (1, 3, 3;\ 1)$, $21 - (0, 6, 2;\ 4)$, $21 - (2, 5, 2;\ 5)$ or $21 - (3, 2, 3;\ 2)$.

3. Construction of a Self-Dual [88, 44, 16] code with an automorphism of type $\mathbf{21-(0,0,4;\ 4)}$. Let now the permutation σ of type $21 - (0, 0, 4;\ 4)$ be an automorphism of $\mathcal{C}$. $F_\sigma(\mathcal{C})$ and $E_\sigma(\mathcal{C})$ are defined as in (4) and (5).

The next proposition follows from [8, Theorems 1–3].

Proposition 2. *Let $\mathcal{C}$ be a self-dual doubly-even code of length* 88 *with an automorphism σ of type* $21 - (0, 0, 4;\ 4)$. *Then,*

(1) *$\mathcal{C} = F_\sigma(\mathcal{C}) \oplus E_\sigma(\mathcal{C})$.*

(2) *$F_\sigma(\mathcal{C})$ and $E_\sigma(\mathcal{C})$ are σ-invariant, that is, invariant under the action of σ.*
(3) *The subcodes $F_\sigma(\mathcal{C})$ and $E_\sigma(\mathcal{C})$ have dimensions 4 and 44 respectively.*
(4) *$\pi(F_\sigma(\mathcal{C}))$ is a self-dual code of length 8.*

The image $\pi(F_\sigma(\mathcal{C}))$ is a binary self-dual [8, 4] code. The only such codes are C_2^4 and A_8. Let $\pi(F_\sigma(\mathcal{C})) = A_8$. Then as a generator matrix of $F_\sigma(\mathcal{C})$ we can consider the following matrix:

$$X = \left(\begin{array}{cccc|cccc} \mathbf{1} & & & & & 1 & 1 & 1 \\ & \mathbf{1} & & & 1 & & 1 & 1 \\ & & \mathbf{1} & & 1 & 1 & & 1 \\ & & & \mathbf{1} & 1 & 1 & 1 & \end{array}\right), \tag{7}$$

where $\mathbf{1}$ is the all-one vector of length 21 and the blanks are zeroes.

Denote by $\mathcal{P}$ the set of even-weight polynomials in the factor-ring $\mathcal{R}_{21} = \mathbb{F}_2[x]/(x^{21}-1)$. The factorization of the polynomial $x^{21}-1$ over the binary field is given by $x^{21}-1 = h_0(x)h_1(x)h_2(x)h_3(x)h_4(x)h_5(x)$, where $h_0(x) = 1+x$, $h_1(x) = 1+x+x^2$, $h_2(x) = 1+x+x^3$, $h_3(x) = 1+x^2+x^3$, $h_4(x) = 1+x+x^2+x^4+x^6$ and $h_5(x) = 1+x^2+x^3+x^4+x^5+x^6$ are irreducible polynomials over $\mathbb{F}_2$.

Let I_j be the ideal of $\mathcal{R}_{21}$ generated by the polynomial $\dfrac{x^{21}-1}{h_j(x)}$. Then I_j is a cyclic code which is isomorphic to the field $\mathbb{F}_2^{\deg h_j(x)}$ for $j = 1,2,3,4,5$ and, moreover, $\mathcal{P} = I_1 \oplus I_2 \oplus I_3 \oplus I_4 \oplus I_5$. The orthogonal idempotent of I_j, $j = 1,\dots,5$ is $\epsilon_j(x) = e_0 + e_1x + e_2x^2 + \cdots + e_{20}x^{20}$, where ϵ_j are:

j	$e_0e_1 \quad \dots \quad e_{20}$
1	011011011011011011011
2	111010011101001110100
3	100101110010111001011
4	011010011001001010000
5	000001010010011001011

As a primitive element of I_j, $j = 1,\dots,5$, we use $\mu_j(x) = m_0 + m_1x + m_2x^2 + \cdots + m_{20}x^{20}$, where μ_j are:

j	$m_0m_1 \quad \dots \quad m_{20}$
1	110110110110110110110
2	100111010011101001110
3	101110010111001011100
4	011011110000101110101
5	010101110100001111011

Using GAP [17], the minimum distance of the cyclic codes $I_1, \ldots, I_5$ is calculated. We obtain that I_1, I_2, I_3, I_4 and I_5 are respectively [21, 2, 14], [21, 3, 12], [21, 3, 12], [21, 6, 8] and [21, 6, 8] codes.

Let $E_\sigma(\mathcal{C})^*$ be the subcode $E_\sigma(\mathcal{C})$ with the last four coordinates deleted. We define the map $\varphi:\ E_\sigma(\mathcal{C})^* \to \mathcal{P}^4$ by identifying the restricted vector $v|\Omega_i = (v_0, v_1, \ldots, v_{20})$ with the polynomial $\varphi(v|\Omega_i)(x) = v_0 + v_1 x + \cdots + v_{20}x^{20}$ for $i = 1,2,3,4$.

From [16, Lemma 6] $\varphi(E_\sigma(\mathcal{C})^*)$ is a self-orthogonal code in $\mathcal{P}^4$ under the inner product $\langle u, v\rangle = \sum_{i=1}^{4} u_i(x)v_i(x^{-1})$. Therefore, we can take a generator matrix for $\varphi(E_\sigma(\mathcal{C})^*)$ of the form

$$Y' = \begin{pmatrix} \epsilon_1(x) & 0 & \alpha_1(x) & \alpha_2(x) \\ 0 & \epsilon_1(x) & \alpha_3(x) & \alpha_4(x) \\ \epsilon_2(x) & 0 & \beta_1(x) & \beta_2(x) \\ 0 & \epsilon_2(x) & \beta_3(x) & \beta_4(x) \\ \beta_1(x^{-1}) & \beta_3(x^{-1}) & \epsilon_3(x) & 0 \\ \beta_2(x^{-1}) & \beta_4(x^{-1}) & 0 & \epsilon_3(x) \\ \epsilon_4(x) & 0 & \gamma_1(x) & \gamma_2(x) \\ 0 & \epsilon_4(x) & \gamma_3(x) & \gamma_4(x) \\ \gamma_1(x^{-1}) & \gamma_3(x^{-1}) & \epsilon_5(x) & 0 \\ \gamma_2(x^{-1}) & \gamma_4(x^{-1}) & 0 & \epsilon_5(x) \end{pmatrix},$$

where $\alpha_i(x) \in I_1$, for $i = 1,2,3,4$, $\beta_i(x) \in I_2$ and $\beta_i(x^{-1}) \in I_3$, $i = 1,2,3,4$, and $\gamma_i(x) \in I_4$, $\gamma_i(x^{-1}) \in I_5$ for $i = 1,2,3,4$, whereas $\epsilon_i(x)$, $i = 1,2,3,4,5$ are defined above.

The corresponding generator matrix of the subcode $E_\sigma(\mathcal{C})^*$ is

$$Y = \begin{pmatrix} y_{1,1} & \cdots & y_{1,4} \\ \vdots & \ddots & \vdots \\ y_{10,1} & \cdots & y_{10,4} \end{pmatrix}, \tag{8}$$

where $y_{i,j}$, $i = 1,2$, $j = 1,\ldots,4$ are right-circulant 2×21 matrices, $y_{i,j}$ for $i = 3,\ldots,6$, $j = 1,\ldots,4$ are right-circulant 3×21 matrices, $y_{i,j}$ for $i = 7,\ldots,10$, $j = 1,\ldots,4$, are right-circulant 6×21 matrices. The first rows of the circulants correspond to the polynomials of the matrix Y'. Thus, we constructed a possible generator matrix of $\mathcal{C}$.

Proposition 3. *Let a binary self-dual doubly-even code $\mathcal{C}$ of length 88 have an automorphism σ of type* $21-(0,0,4;\ 4)$. *Then a possible generator matrix* *of $\mathcal{C}$ can be written as*

$$G = \left(\begin{array}{c|c} \multicolumn{2}{c}{X} \\ \hline & 0000 \\ Y & \vdots \vdots \vdots \vdots \\ & 0000 \end{array} \right), \tag{9}$$

where X and Y are defined in (7) and (8).

A computer check shows that many self-dual doubly-even codes with a generator matrix of the type (9) are extremal. Here we present 35 examples $\mathcal{C}_1$, $\mathcal{C}_2$, ..., $\mathcal{C}_{35}$ of extremal codes. To define completely their generator matrices G_1, ..., G_{35}, it is sufficient to give the submatrix Y of G in (9). The matrix Y is determined by the circulant matrices $y_{i,j}$, $i = 1, \ldots, 12$, $j = 1, \ldots, 6$ whose first rows are vectors corresponding to polynomials of the matrix Y'. The values of the polynomials in Y' for the codes $\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_{35}$ are as follows: $\alpha_1(x) = \alpha_4(x) = 0$, $\alpha_2(x) = \alpha_3(x) = \mu_1(x)$; $\beta_1(x) = 0$, $\beta_i(x)$ is 0 or $\mu_2^{t_i}(x)$ for $i = 2, 3, 4$ and $t_i = 1, \ldots, 7$; $\gamma_1(x) = \epsilon_4(x)$, $\gamma_i(x)$ is 0 or $\mu_4^{s_i}(x)$ for $i = 2, 3, 4$ and $s_i = 1, \ldots, 63$. The values of the degrees t_i and s_i for $i = 2, 3, 4$ are listed in Table 1. We note that if the value of $\gamma_i(x)$ or $\beta_i(x)$ is 0, then the corresponding entry for t_i or s_i is empty.

The weight enumerator of an extremal doubly-even self-dual [88,44,16] code is uniquely determined [4]:

$$W_C = 1 + 32164y^{16} + 6992832y^{20} + 535731625y^{24} + 16623384448y^{28} + 225426781470y^{32} + \cdots$$

To prove the inequivalence of the codes we use the same invariants as in [6] and [5]. Let M be the set of all 32164 codewords of weight 16 and $A_{i,j}$ be the number of the codewords of M that have one at the coordinate positions i and j. It is clear that the set of numbers $\{A_{i,j} | \ 1 \leq i < j \leq 88\}$ is an invariant for equivalent codes. So, the smallest and the largest element $m(2)$ and $M(2)$, respectively, in the set are invariants as well.

The values of $m(2)$ and $M(2)$ for the codes $\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_{35}$ are listed in Table 1.

Table 1 implies that the presented new 35 extremal self-dual codes of length 88 are inequivalent and, moreover, together with the data in [7] and [5] it follows that these codes and the codes given in [6] and [5] are inequivalent as well.

Table 1. Matrices Y' and invariants

$Code$	t_2	t_3	t_4	s_2	s_3	s_4	$M(2)$	$m(2)$
$\mathcal{C}_1$	1	1	1	63	1	21	1071	672
$\mathcal{C}_2$	1	1	1	3	2	30	1080	819
$\mathcal{C}_3$	1	1	1	3	1	27	1089	777
$\mathcal{C}_4$	5	3	1		1	28	1092	756
$\mathcal{C}_5$	7	1	1		2	42	1095	714
$\mathcal{C}_6$	1	1	1		1	54	1098	714
$\mathcal{C}_7$	1	1	1	3	2	39	1101	777
$\mathcal{C}_8$	1	1	1		2	26	1104	777
$\mathcal{C}_9$	1	1	1	3	1	3	1107	801
$\mathcal{C}_{10}$	7	1	1		2	19	1110	864
$\mathcal{C}_{11}$	7	1	1		2	11	1113	738
$\mathcal{C}_{12}$	7	7	1		1	26	1113	777
$\mathcal{C}_{13}$	1	1	1		1	6	1116	672
$\mathcal{C}_{14}$	7	7	1		1	25	1131	861
$\mathcal{C}_{15}$	1	1	1	3	1	20	1134	819
$\mathcal{C}_{16}$	1	1	1	3	1	45	1137	777
$\mathcal{C}_{17}$	1	1	1	1	1	5	1152	882
$\mathcal{C}_{18}$	1	1	1		1	8	1155	630
$\mathcal{C}_{19}$	1	1	1		1	21	1158	780
$\mathcal{C}_{20}$	7	1	1		2	62	1176	630
$\mathcal{C}_{21}$	1	1	1		1	52	1179	735
$\mathcal{C}_{22}$	1	1	1		2	23	1197	693
$\mathcal{C}_{23}$	7	1	1		2	61	1218	717
$\mathcal{C}_{24}$	7	7	1		1	12	1221	903
$\mathcal{C}_{25}$	1	1	1		2	30	1239	693
$\mathcal{C}_{26}$	7	1	1		3	1	1242	840
$\mathcal{C}_{27}$	7	1	1		2	59	1263	885
$\mathcal{C}_{28}$	7	1	1		2	21	1281	735
$\mathcal{C}_{29}$	5	3	1		1	12	1302	756
$\mathcal{C}_{30}$	1	1	1		1	55	1323	612
$\mathcal{C}_{31}$	7	1	1		2	57	1344	843
$\mathcal{C}_{32}$	1	1	1	63	1	37	1347	798
$\mathcal{C}_{33}$	1	1	1	3	1	21	1365	840
$\mathcal{C}_{34}$	7	1	1		3	8	1368	840
$\mathcal{C}_{35}$	1	1	1	3	1	6	1389	861

Theorem 4. *Up to equivalence there are at least 105 binary extremal self-dual doubly-even codes of length 88, where 35 are new.*

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