Serdica J. Computing **3** (2009), 205–226

Serdica
Journal of Computing

Bulgarian Academy of Sciences
Institute of Mathematics and Informatics

THE VALUE OF KNOWING THAT YOU DO NOT KNOW

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Abstract. The value of knowing about data availability and system accessibility is analyzed through theoretical models of Information Economics. When a user places an inquiry for information, it is important for the user to learn whether the system is not accessible or the data is not available, rather than not have any response. In reality, various outcomes can be provided by the system: nothing will be displayed to the user (e.g., a traffic light that does not operate, a browser that keeps browsing, a telephone that does not answer); a random noise will be displayed (e.g., a traffic light that displays random signals, a browser that provides disorderly results, an automatic voice message that does not clarify the situation); a special signal indicating that the system is not operating (e.g., a blinking amber indicating that the traffic light is down, a browser responding that the site is unavailable, a voice message regretting to tell that the service is not available). This article develops a model to assess the value of the information for the user in such situations by employing the information structure model prevailing in Information Economics. Examples related to data accessibility in centralized and in distributed systems are provided for illustration.

ACM Computing Classification System (1998): H.4.2, H.3.3.
Key words: Information Economics, Information Structures, Blackwell Theorem, Knowledge Management, Data Retrieval, System Accessibility, Data Availability.

The results of the analysis indicate that there is a direct relationship between the knowledge about systems accessibility or data availability and its informativeness. The addition of a signal indicating system inaccessibility or data unavailability increases the expected value of information derived from the system. The article proves a theorem related to the research question and discusses the theoretical and practical interpretation of the results.

1. Introduction.

Despite the ever-increasing importance of information resources, evaluating the actual benefit of using information systems remains problematic. The normative models that aim to forecast the real value of information cannot always be related directly or unequivocally to the real value of the information systems [3]. Moreover, over the years significant research has been conducted to address issues of "not knowing", for example the phenomenon that Simon [26] termed "bounded rationality". Some of its aspects were analyzed comprehensively by Rubinstein [23].

In this study we attempt to model the value of knowing whether the user can obtain requested data or not. Specifically, we evaluate the benefits deriving from adding a feature to a system indicating the unavailability of information. This discussion is specifically relevant to the design of Data Mining and Knowledge Management systems, Browsers, and Decision Support Systems.

To simplify the discussion, we begin by defining a number of key concepts.

Level of accessibility: the percentage of queries that the information system responds to, based on accessible information, providing the user with the requested information.

Absolute uncertainty, no-information: a decision situation in which the information system user does not possess any additional information beyond the a priori probability of states of nature occurring.

No-information signal: a special signal received in situations where the system is not accessible.

The methodology of the study. We will describe situations of lack of information by extending the information structure model [19, ch. 5.] We will investigate the ***quantitative*** value of knowing that there is no accessible information, as opposed to a situation in which the user does not know that the information is not accessible. The value of the information of various models expressing system inaccessibility will be compared analytically.

An example of a real life situation is the blinking amber signal, which tells us that the traffic light is out of order. Closer to the subject of our discussion is the Windows hourglass, which indicates that the operating system is busy and the results of its operations are not yet available. Another example, of course, is a browser's response that a requested website is not accessible. This is a no-information signal, without which we do not know if the system is accessible or not. We will show that an information system that produces a no-information signal to indicate unavailability is generally more informative than any similar information system that indicates unavailability in a different way. Clearly, the usefulness of an information system to the user is seriously restricted if the user cannot identify situations of unavailability.

The structure of the article. The next section reviews the pertinent literature in the area of Information Economics. Moreover, it focuses on the information structure model and the Blackwell Theorem [19, ch. 5]. Section 3 describes the factors that affect the value of the information, and discusses the basic premises underlying the model. It then describes the models in the light of these underlying premises. Section 4 presents examples that illustrate the general informativeness ratios and the value of information systems under different levels of accessibility. A theorem that characterizes the value of a no-information signal is presented and illustrated. Section 5 draws some conclusions and discusses the contribution of the study and its significance.

2. Analytical research into the value of information in the area of Information Economics. In assigning an expected normative economic value to information systems, many researchers made use of Microeconomic theory and Statistical Decision Theory tools. The combination of utility theory and the perception of a noisy system led to the construction of a probabilistic statistical model that accords to an information system the property of transferring input data (states of nature) to output (signals) with a certain statistical probability [16, 18]; see also the collection of articles on the subject edited by McGuire and Radner [19]. This model, which delineates a noisy information system, is called the information structure model. It is based on the assumption that the system is noisy but it does not analyze the nature of the noise. In this study, we analyze the noise and construct a model that attempts to deal with two types of noise that are typical to an information system:

1) *An information system that produces a no-information signal in a situation of absence of information, namely, a system that informs the user that it*

cannot produce a meaningful signal in certain situations. The user knows that he or she does not know. (For example, when someone asks me for Julia Roberts' home telephone number I can say immediately that I do not know it. The decision-maker, that is the person requesting the information, knows that he or she has received a no-information signal.)

2) *An information system that produces a random signal in the case of lack of information.* The user does not know that he or she does not know (for instance, in continuation of the previous example, if the answer is a series of random digits in the form of a telephone number, the decision-maker does not know that he or she does not know).

   Intuitively, receiving a no-information signal should be better than receiving a random signal, but it might also incur some cost due to the need of adding a function to the system. So how can a decision-maker evaluate the benefit of knowing that he or she does not know?

In this study, we will address the issue of not knowing by finding tools to determine a general informativeness ratio amongst information systems. We will build models to represent lack of information and present general informativeness ratios among different levels of lack of information.

2.1. The information structure model and the Blackwell Theorem. The tool employed to investigate the phenomena described in section 4 is the information structure model [19, ch. 5, pp. 101–109], [5]. This is a general model for comparing information systems based on the assumptions of rational behavior. According to the information structure model, four factors determine the expected value of information.

1. The a priori probabilities of a set of pertinent states of nature.
2. The information system – a stochastic (Markovian) matrix that transmits states of nature to signals.
3. The decision matrix – a stochastic matrix that links the signals with the decision set of the decision-maker.
4. The payoff matrix – a matrix that presents the quantitative compensation to the decision-maker resulting from the combination of a decision chosen and a given state of nature.

The information structure model enables comparison of information systems in terms of their quantitative economic value. An information structure Q_1 is said to be more informative than an information structure Q_2 if the expected payoff of using Q_1 is no lower than the expected payoff of using Q_2. Two information structures may be compared at several levels:

1. Comparison between two information structures for a given vector of a priori probabilities and a given payoff matrix.

2.1. Comparison between two information structures when the vector of probabilities for the occurrence of states of nature is given and all payoff matrices are possible (a generalization for all possible compensations).

2.2. Comparison between two information structures when the payoff matrix is known in advance and the vector of a priori probabilities of states of nature is not known in advance (a generalization for all possible probabilities).

3. Comparison between two information structures for all vectors of a priori probabilities of the occurrence of states of nature and any possible payoff matrices.

When information structure Q_1 is more informative than information structure Q_2 according to ratio 3 above (irrespective of compensations and a priori probabilities), a ***general informativeness ratio*** is defined between them. This ratio and the conditions for its existence are defined in Blackwell's Theorem [19, ch. 5].

Let us now consider a model in which S is a finite set of n states of nature: $S = \{S_1, \ldots, S_n\}$.

Let

$$\Pi = \begin{pmatrix} p_1 & 0 & \ldots & 0 & 0 \\ 0 & p_2 & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & \ldots & 0 & p_n \end{pmatrix},$$

where Π is a square matrix whose diagonal is the a priori probabilities of obtaining states of nature and the remaining elements are equal to 0.

Let A be a finite set of k possible decisions, $A = \{A_1, \ldots, A_k\}$.

Let U be the payoff function: $U : A \times S \rightarrow \Re$ (a combination of a state of nature and a decision gives a fixed compensation that is a real number).

Let Y be a finite set of m signals, $Y = \{Y_1, \ldots, Y_m\}$.

An **information structure** Q is defined such that its elements obtain values between 0 and 1,

$$Q : S \times Y \to [0, 1].$$

$Q_{i,j}$ is the probability that a state of nature S_i displays a signal Y_j.

$$\sum_{j}^{m} Q_{i,j} = 1.$$

Let D be the decision function. Like Q, D is a stochastic (Markovian) matrix; namely, it is assumed that the decision is not necessarily always the same for a given signal.

$$D : Y \times A \to [0, 1].$$

The expected payoff is $\mathrm{Trace}(\Pi * Q * D * U)$, where Trace is an operator that sums up the diagonal terms of the square matrix. The objective function for maximizing the compensation expectation is $\underset{D}{\mathrm{Max}}\,(\mathrm{trace}(\Pi * Q * D * U))$.

When the utility function is linear, that is, the decision-maker is of EMV (Expect Monetary Value) type [20], a linear programming problem is obtained, where the variables being the elements of the decision matrix D. One of the optimal solutions is in a form of a decision matrix whose elements are 0 or 1 (a pure decision).

A numerical example: Let Q be an information structure representing an information system for managing a firm's inventory: when an item is requested, the system has to say whether it is in stock. For the sake of simplicity, suppose there is only one item in stock. Thus there are two states of nature:

S_1 – the item is in stock $p(S_1) = 0.9$;
S_2 – the item is not in stock, $p(S_2) = 0.1$.

The information system produces two signals:

Y_1 – the item is available;
Y_2 – the item is not available.

Suppose $Q = \begin{pmatrix} 0.95 & 0.05 \\ 0.05 & 0.95 \end{pmatrix}$ is this information structure.

Let A be the set of possible decisions: A_1 – do not order; A_2 – order. Let D be the decision matrix: $D = \begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix}$.

Let U be the payoff matrix: $U = \begin{pmatrix} 0 & -5 \\ -1 & 0 \end{pmatrix}$.

Note: -1 indicates a loss due to ordering an item that is in stock (cost of maintaining stock); -5 indicates a loss due to not ordering an item that is not in stock (non-production cost).

The probability matrix Π is described as follows: $\Pi = \begin{pmatrix} 0.9 & 0 \\ 0 & 0.1 \end{pmatrix}$

It can be shown that $\underset{D}{\text{Max}}\,(\text{trace}(\Pi * Q * D * U)) = -0.07$, where $D = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is the decision matrix, which contains two pure decision rules (A_1 – Do not order | Y_1; A_2 – Order | Y_2).

Over the years, a number of researchers developed analytical models to implement the concept of the "Information structure model" in order to evaluate the value of information. Some studies analyzed the correlation between this method of operation research and information technology. Ahituv [2] demonstrated the life cycle of decision support information system with the model. Ahituv and Elovici [4] evaluated the value of the performances of distributed information systems. Elovici et al. [15] used this method to compare performances of Information Filtering Systems. Ahituv and Greenstein [5] used this model to assess issues of centralization vs. decentralization. Aronovich and Spiegler [10] use this model in order to assess the effectiveness of data mining processes.

Moreover, the model was expanded to evaluate the value of information in several aspects: the value of a second opinion [6], the value of information nonlinear models of the utility theory [24], analyzing the situation of case dependent signals (the set of signal is dependent on the state of nature, [28]), a situation of a two-criteria utility function [22].

The model was applied to evaluate empirically the value of information in a post office [14] and in the process of analysis of Quality Control methods [21, 17].

2.2. A convex combination of information structures. Sulganik [27] indicates that a convex combination of information structures could be used to describe experimental processes (with a probability p of success and $(1-p)$ of failure). For example, he investigates the convex combination of two information structures: one presents perfect information and the other one no-information (all its rows are identical).

The mechanism of convex combinations of information structures is employed in an earlier research by Ahituv and Greenstein [5] which analyses the effect of probabilistic availability of information systems on productivity and illuminates some aspects of the phenomenon termed "the productivity paradox" [11, 12, 13, 8]. In this paper we will use this mechanism to analyze quantitatively the value of being aware of the unavailability of information.

A convex combination of two information structures is defined as follows:

Let Q_1 and Q_2 be two information structures describing information systems. Let $S = \{S_1, \ldots, S_n\}$ be their set of the states of nature. Let $Y = \{Y_1, \ldots, Y_m\}$ be their set of signals. When a decision situation is given let p be the probability that Q_1 will be activated, and $(1-p)$, that Q_2 will be activated. Q_3, the weighted information structure, is represented by a convex combination of Q_1 and Q_2

$$Q_3 = p * Q_1 + (1-p) * Q_2.$$

The decision maker is not aware, of course, which information structure is activated.

2.3. The general informativeness ratio. Given two information systems that deal with the same state of nature and are represented by the information structures Q_1 and Q_2, Q_1 will be considered generally more informative than Q_2 if its expected payoff is no lower than that of Q_2 for all a priori probability vectors and any payoff matrix. In terms of the information structure model, if for every possible payoff matrix U and for every a priori probability matrix Π $\underset{D}{\text{Max}}\,(\text{trace}(\Pi * Q_1 * D * U)) \geq \underset{D}{\text{Max}}\,(\text{trace}(\Pi * Q_2 * D * U))$ (where trace is the sum of all the elements of the main diagonal of the product square matrix), then Q_1 is generally more informative than Q_2, denoted $Q_1 \geq Q_2$. Blackwell's Theorem states that Q_1 is generally more informative than Q_2 if and only if there is a Markovian (stochastic) matrix R such that $Q_1 * R = Q_2$. It should be noted that the general informativeness ratio is a partial rank ordering. There is no necessary rank order between any two information structures. The rank ordering is transitive.

A numerical example:

$$Q_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \geq Q_2 = \begin{pmatrix} 0.95 & 0.05 \\ 0.05 & 0.95 \end{pmatrix},$$

and

$$Q = \begin{pmatrix} 0.95 & 0.05 \\ 0.05 & 0.95 \end{pmatrix} \geq Q_3 = \begin{pmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{pmatrix},$$

Hence $Q_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \geq Q_3 = \begin{pmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{pmatrix}$.

3. Modeling accessibility level by using the Information Structure Model. In this section we will discuss the notion of the accessibility of an information system.

3.1. Level of accessibility. Let p be the probability that an information system will provide an effective response to a query. An effective response is obtained if the system is accessible for a period of time sufficient for making a decision, and if the system can provide the user with the information required.

3.2. The effect of a signal produced by the system when it is not accessible. IS professionals can design fault-handling scenarios for situations in which the system is not available. There is a possibility of acknowledging users about situations of unavailability by alarming the decision-makers with a special no-information signal [5]. This paper discusses and analyzes two scenarios related to inaccessibility:

1) The system randomly produces a signal of some sort. The user is not aware of the system's inaccessibility.

2) The system produces a no-information signal indicating that the system is not accessible. Thus, the user is aware that the system is not accessible.

3.3. Typical information structures. In order to analyze the situations of lack of information, various types of information structures are demonstrated. These types of information structures react differently to situations of inaccessibility. Some of them deal with total uncertainty, which mean that they do not produce any meaningful signal. Other types of information structures inform us using a special signal that the information is not available. The analysis of the level of accessibility will be introduced in the following subsection. First, some information structures that pertain to an inaccessibility situation are presented.

3.3.1. A general information structure representing total uncertainty. Let $M(Q)$ be an information structure describing a situation of total absence of information for a set of states of nature S and a set of signals Y of the information structure Q, such that

$$\forall\, i,\ 1 \le i \le n \quad \forall\, j,\ 1 \le j \le m \quad M(Q)_{i,j} = P(Y_j|S_i)$$

$$\forall\, i,\ 1 \le i \le n \quad \forall\, k,\ 1 \le k \le n \quad \forall\, j,\ 1 \le j \le m \quad M(Q)_{i,j} = M(Q)_{k,j}$$

Such information structure implies that it is not possible to have any additional information about the event that had triggered a given signal beyond what can be elicited from the a-priori probabilities.

Example: $0 \le a \le 1$ $Y = \{Y_1, Y_2\}$ $S = \{S_1, S_2\}$ $M(Q) = \begin{pmatrix} a & 1-a \\ a & 1-a \end{pmatrix}$.

In subsections 3.3.2, and 3.3.3 two special cases of $M(Q)$ will be presented.

3.3.2. An information structure representing uncertainty – a random signal with uniform distribution. Let $N(Q)$ be an information structure describing a situation of absolute uncertainty for a set of states of nature S and a set of signals Y of an information structure Q, namely, a random signal is received in a situation of uncertainty. $N(Q)$ is a special case of an information structure of the type $M(Q)$, which represents a situation of absence of information. It is described as follows:

$$\forall\, i,\ i = 1, \ldots, n \quad \forall\, j,\ j = 1, \ldots, m : N(Q)_{i,j} = P(Y_j|S_i) = 1/m.$$

Obviously, for all i: $\sum_{j=1}^{m} = N(Q)_{i,j} = 1$. For example, $Y = (Y_1, Y_2)$, $S = (S_1, S_2)$, $N(Q) = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}$.

3.3.3. An information structure representing an information system that identifies lack of information, and an information structure representing the display of a no-information signal. Let Q be an information structure describing an information system. Let $S = \{S_1, \ldots, S_n\}$ be the set of the states of nature of Q. Let $Y = \{Y_1, \ldots, Y_m\}$ be the set of signals of Q. Let Y_{m+1} be a no-information signal which is not a part of the set of signals of Q.

Clarification. Y_{m+1} is a signal that is not displayed during normal system operations, but only when a situation of absence of information is identified. This signal aims to indicate cases in which the system is not accessible for the user and identifies itself as such. In other words, the system informs the user that it is not informative at that moment.

Let Q_0 be an information structure describing an information system with the addition of a no-information signal. Let $S = \{S_1, \dots, S_n\}$ be the set of the states of nature of Q_0. Let $Y_0 = \{Y_1, \dots, Y_m, Y_{m+1}\}$ be the set of signals of Q_0.

$$\begin{array}{lll} Q0_{i,j} = Q_{i,j} & 1 \le i \le n, & 1 \le j \le m \\ Q0_{i,j} = 0 & 1 \le i \le n, & j = m+1. \end{array}$$

The levels of general informativeness of Q and Q_0 are identical [1]. Therefore Q_0 will be used later to represent the information structure Q in the models illustrating information systems that provide a no-information signal.

Let $N_0(Q)$ be a structure that represents a no-information signal for information structure Q. Let $S = \{S_1, \dots, S_n\}$ be the set of the states of nature of Q. Let $Y = \{Y_1, \dots, Y_m\}$ be the set of signals of Q. Let Y_{m+1} be a no-information signal that is not part of the set of signals of Q. Let S be its set of states of nature and $Y_0 = \{Y_1, \dots, Y_m, Y_{m+1}\}$ the set of signals of $N_0(Q)$. $N_0(Q)_{i,j} = 0$, where $0 \le j \le m$ and $N_0(Q)_{i,j} = 1$, where $j = m+1$.

Example. $S = \{S_1, S_2\}$, $Y = \{Y_1, Y_2\}$, $Y_0 = \{Y_1, Y_2, Y_3\}$; Y_3 is the no-information signal, $N_0(Q) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$.

3.4. Accessibility in Terms of Probability. Suppose that p is the weighted level of accessibility of Q_1. We denote the actualization of the information structure Q_1 at a level of accessibility p by Q_2. Q_2 is the convex combination of Q_1 and $M(Q_1)$.

$$Q_2 = p * Q_1 + (1 - p) * M(Q_1)$$

Q_1 is generally more informative than Q_2 [5, Theorem 1].

4. The value of the awareness to the unknown. Two scenarios are demonstrated, and analyzed in this section; first we analyze the case when the user does not know that the system is inaccessible (sub-section 4.1); second, in contrast, when the user knows that the system is not accessible (sub-section 4.3).

4.1. Producing a uniformly distributed signal in a situation of uncertainty – an example. We first illustrate the case by a using the example of Q, an information structure for managing a firm's inventory, that was shown earlier in Sub-Section 2.1. Suppose the system produces a random signal in a no-information situation. Table 4.1 exhibits the results for three levels of accessibility. It is assumed that when the system is not accessible, a uniformly distributed random signal is produced.

Table 4.1. Receipt of a Uniformly Distributed Signal in Situations of Inaccessibility

Level of Accessibility p	100%	90%	80%
Information structure	$Q = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$		
The weighted information structure $Q_i = p_i * Q + (1 - p_i) * N(Q)$	$Q_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$Q_2 = \begin{pmatrix} 0.95 & 0.05 \\ 0.05 & 0.95 \end{pmatrix}$	$Q_2 = \begin{pmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{pmatrix}$
The matrix of a priori probabilities for the states of nature	$\Pi = \begin{pmatrix} 0.9 & 0 \\ 0 & 0.1 \end{pmatrix}$		
The payoff matrix	$U = \begin{pmatrix} 0 & -5 \\ -1 & 0 \end{pmatrix}$		
Optimal decision matrix (A_1 – do not order, A_2 – order)	$D = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$D = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$D = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
expected payoff $\underset{D}{\text{Max}}\,(\text{trace}(\Pi * Q_i * D * U))$	0	−0.07	−0.14

4.2. The value of a no-information signal in probabilistically accessible systems. Consider now two information systems with the same level of accessibility, and identical performance when they are accessible. It is intuitively reasonable to expect that an information system that always produces a no-information signal when it has ceased to be accessible would be generally more informative than any “similar” system that does not produce a no-information signal in situations of inaccessibility. This is proved in Theorem 1.

Theorem 1.

1. *Let Q be an information structure describing an information system. Let $S = \{S_1, \ldots, S_n\}$ be the set of states of nature of Q. Let $Y = \{Y_1, \ldots, Y_m\}$ be the set of signals of Q.*

2. *Let Y_{m+1} be a no-information signal that is not a part of the set of signals of Q. Let Q_0 be an information structure describing an information system with the addition of a no-information signal. Let $S = \{S_1, \ldots, S_n\}$ be the set of states of nature of Q_0. Let $Y_0 = \{Y_1, \ldots, Y_m, Y_{m+1}\}$ be the set of signals of Q_0.*

3. *Let $M(Q)$ be a structure that represents uncertainty. Let S be the set of its states of nature and Y its set of signals.*

   $$\forall\, i,\ 1 \le i \le n \quad \forall\, k,\ 1 \le k \le n\ \forall\, j,\ 1 \le j \le m,\ M(Q)_{i,j} = M(Q)_{k,j} = m_j$$

   $M(Q)$ is an information structure whose rows are all identical, m_j being an element that appears in all the rows of the j column.

4. *Let $M_0(Q)$ be the information structure $M(Q)$ with the addition of a no-information signal (column of zeros). Let S be the set of its states of nature and Y_0 its set of signals.*

5. *Let $N_0(Q)$ be a structure that represents a display of a no-information signal. Let S be the set of its states of nature and Y_0 its set of signals.*

   $$N_0(Q)_{i,j} = 0 \quad \textit{when} \quad 1 \le j \le m \quad N_0(Q)_{i,j} = 1 \quad \textit{when} \quad j = m + 1.$$

6. *Let p be the accessibility of the system, $0 \le p \le 1$*

7. *Let Q_1 be a convex combination of Q_0, $M_0(Q)$ and $N_0(Q)$*

   $$Q_1 = p * Q_0 + p_1 * M_0(Q) + p_2 * N_0(Q), \quad 0 \le p_1,\ 0 \le p_2,\ p_1 + p_2 = 1 - p.$$

8. *Let Q_2 be a convex combination of Q_0 and $N_0(Q)$*

   $$Q_2 = p * Q_0 + (1 - p) * N_0(Q).$$

Then Q_2 is generally more informative than Q_1.

The proof is shown in the appendix.

Theorem 1 asserts that an information system that always produces a no-information signal when the system or the data are not accessible is generally more informative than any "similar" system that is accessible at an identical level of accessibility, but does not produce a no-information signal when the system or the data are not accessible. Consequently, it is more valuable for an information system to indicate situations of inaccessibility to the user.

4.3. Producing a no-information signal – an example. Consider the information structure Q of the previous example in a similar situation with the only difference that there exists a no-information signal. Let Y_3 be a signal that does not belong to the set of signals Y. Y_3 denotes that the system is not functioning at the moment.

Let Q_0 be an information structure representing the information system with the addition of a no-information signal Y_3. $Q_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$.

Q and Q_0 are identical in terms of general informativeness [1]. Let us now examine a number of examples of using the system by means of various data configurations (the numerical examples are similar to those given in example 4.1, apart from the addition of a no-information signal). Table 4.3 shows that the lower the level of accessibility, the lower the expected payoff provided from using the information system. The table exhibits the results of three levels of accessibility assuming that when the system is not accessible, a no-information signal is produced.

Table 4.3. A Case of a No-information Signal in Situations of Inaccessibility

Level of Accessibility p	100%	90%	80%
Information structure	$Q = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$		
The weighted information structure $Q_{i-0} = p_i * Q_0 + (1 - p_i) * N_0(Q)$	$Q_{1-0} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$	$Q_{2-0} = \begin{pmatrix} 0.9 & 0 & 0.1 \\ 0 & 0.9 & 0.1 \end{pmatrix}$	$Q_{3-0} = \begin{pmatrix} 0.8 & 0 & 0.2 \\ 0 & 0.8 & 0.2 \end{pmatrix}$
The matrix of a priori probabilities for the states of nature	$\Pi = \begin{pmatrix} 0.9 & 0 \\ 0 & 0.1 \end{pmatrix}$		
The payoff matrix	$U = \begin{pmatrix} 0 & -5 \\ -1 & 0 \end{pmatrix}$		
Optimal decision matrix (A_1 – do not order, A_2 – order)	$D = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$	$D = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$	$D = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$
expected payoff $\max_D (\mathrm{trace}(\Pi * Q_i * D * U))$	0	−0.05	−0.1

Figure 1 portrays a comparison between the expected payoffs as a function of the accessibility level for an information system that produces a uniformly distributed random signal in lack of information situations (as presented in Table 4.1), and a system that produces a no-information signal in such cases (Table 4.3). A comparison of the levels of general informativeness of the two systems at identical accessibility levels indicates that the system producing a no-information signal is generally more informative than the system producing a random signal.

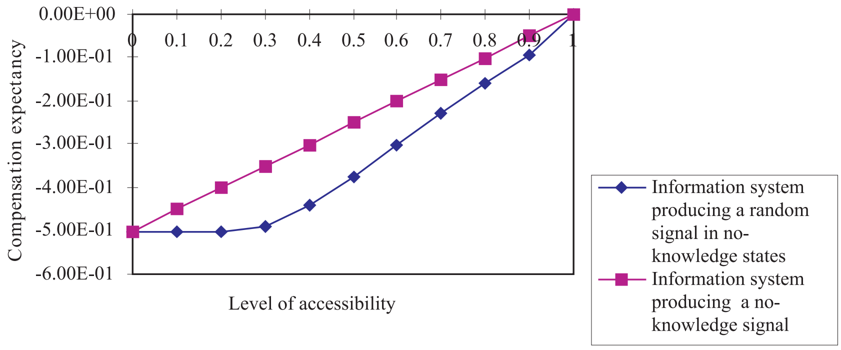

Fig. 1. Comparison of expected payoff for information systems that related differently to no-knowledge situations

5. Summary and Conclusions. The extension of the information structure model by introducing structures to describe no-information situations has enabled us to draw the following conclusions.

There is a direct relationship between the accessibility of the system and the level of general informativeness. The higher the level of accessibility of an information system, the more informative it is generally.

A no-information signal in situations of inaccessibility always increases the level of general informativeness. An information system that tells the user that it is not available at a certain moment is better for the user than a similar system (in terms of the quality of its signaling) that does not have this feature (Theorem 1).

5.1. Significance of the results. The value of an information system as a function of the information system's accessibility was analyzed here. Insofar as system accessibility and data accessibility affect the level of system informativeness, the system configuration, which determines the level of accessibility, has a direct impact on the quality of the informativeness of the system. Therefore, it is advisable to indicate situations of inaccessibility to the user. If an information system does not identify a situation of inaccessibility the benefit derived from it is limited. This conforms to everyday intuitive solutions such as a flashing amber traffic light, a computer that requests the user to wait for a response, a browser that indicates that a requested site is not available, and the like.

5.2. A practical interpretation. Though this study displays a theoretical model, it has also some practical implications. At the stage of examining technical alternatives for implemenring an information system, and particularly when making a configuration decision (e.g., centralization vis-a-vis various forms of decentralization), the level of accessibility of an information system has to be estimated for each possible alternative. The expected level of accessibility should be an additional criterion, because of its direct effect on the value of the information.

In order to improve the possible benefits of an information system, the designers should use a no-information signaling mechanism in inaccessibility situations. At the acceptance-testing stage, the users have to check that in situations of inaccessibility the system does indeed produce a no-information signal. In an evaluation process of several alternative information systems offered to an organization, decision makers must take into account the manner of operation in situations of inaccessibility.

Appendix

Theorem 1.

1. *Let Q be an information structure describing an information system. Let $S = \{S_1, \dots, S_n\}$ be the set of states of nature of Q. Let $Y = \{Y_1, \dots, Y_m\}$ be the set of signals of Q.*

2. *Let Y_{m+1} be a no-information signal that is not a part of the set of signals of Q. Let Q_0 be an information structure describing an information system with the addition of a no-information signal. Let $S = \{S_1, \dots, S_n\}$ be the set of states of nature of Q_0. Let $Y_0 = \{Y_1, \dots, Y_m, Y_{m+1}\}$ be the set of signals of Q_0.*

3. *Let $M(Q)$ be a structure that represents uncertainty. Let S be the set of its states of nature and Y its set of signals.*

$$\forall\ i,\ 1 \le i \le n\ \forall\ k,\ 1 \le k \le n\ \forall\ j,\ 1 \le j \le m,\ M(Q)_{i,j} = M(Q)_{k,j} = m_j$$

$M(Q)$ is an information structure whose rows are all identical, m_j being an element that appears in all the rows of the j column.

4. *Let $M_0(Q)$ be the information structure $M(Q)$ with the addition of a no-information signal (column of zeros). Let S be the set of its states of nature and Y_0 its set of signals.*

5. *Let $N_0(Q)$ be a structure that represents display of a no-information signal. Let S be the set of its states of nature and Y_0 its set of signals.*

   $$N_0(Q)_{i,j} = 0 \quad when \quad 1 \leq j \leq m \qquad N_0(Q)_{i,j} = 1 \quad when \quad j = m + 1.$$

6. *Let p be the accessibility of the system, $0 \leq p \leq 1$*

7. *Let Q_1 be a convex combination of Q_0, $M_0(Q)$ and $N_0(Q)$*

   $$Q_1 = p * Q_0 + p_1 * M_0(Q) + p_2 * N_0(Q), \quad 0 \leq p_1,\ 0 \leq p_2,\ p_1 + p_2 = 1 - p.$$

8. *Let Q_2 be a convex combination of Q_0 and $N_0(Q)$*

   $$Q_2 = p * Q_0 + (1 - p) * N_0(Q).$$

Then Q_2 is generally more informative than Q_1.

Before proving the theorem, a lemma is proved.

Lemma 1.1.

1. *Let Q be an information structure describing an information system. Let $S = \{S_1, \ldots, S_n\}$ be the set of states of nature of Q. Let $Y = \{Y_1, \ldots, Y_m\}$ be the set of signals of Q.*

2. *Let Y_{m+1} be a no-information signal that is not part of the set of signals of Q. Let Q_0 be an information structure describing an information system with the addition of a no-information signal. Let $S = \{S_1, \ldots, S_n\}$ be the set of states of nature of Q_0.*

   Let $Y_0 = \{Y_1, \ldots, Y_m, Y_{m+1}\}$ be the set of signals of Q_0.

3. *Let $M(Q)$ be a structure representing uncertainty. Let S be its set of states of nature and Y its set of signals.*

   $$\forall\ i,\ 1 \leq i \leq n\ \forall\ k,\ 1 \leq k \leq n\ \forall\ j,\ 1 \leq j \leq m, \quad M(Q)_{i,j} = M(Q)_{k,j} = m_j,$$

   $M(Q)$ is an information structure with rows that are all identical, m_j is an element that appears in all the rows of the j column.

4. *Let $N_0(Q)$ be an information structure representing display of a no-information signal. Let S be the set of its states of nature and Y_0 its set of signals.*

$$N_0(Q)_{i,j} = 0 \quad when \quad 1 \le j \le m$$

$$N_0(Q)_{i,j} = 1 \quad when \quad j = m+1.$$

5. $0 \le p \le 1$, *p is the accessibility of the system.*

6. *Let Q_1 be a convex combination of Q and $M(Q)$.*

$$Q_1 = p * Q + (1-p) * M(Q).$$

7. *Let Q_2 be a convex combination of Q_0 and $N_0(Q)$.*

$$Q_2 = p * Q_0 + (1-p) * N_0(Q).$$

Then, Q_2 is generally more informative than Q_1.

Proof of Lemma 1.1. The proof will use the second condition of Blackwell's Theorem in order to show that a specific stochastic matrix R, $Q_1 = Q_2 * R$, can be built. Hence, Q_2 is generally more informative than Q_1.

(1) Let R be a stochastic matrix with dimensions $m+1 \times m$.

$$R_{1i,j} = \begin{cases} 0, & 1 \le i \le m,\ i \ne j \\ 1, & 1 \le i \le m,\ i = j \\ m, & i = m+1 \end{cases} \qquad R_1 = \begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & 1 \\ m_1 & & \dots\dots & & m_m \end{pmatrix}.$$

(2) We observe $Q_3 = Q_2 * R$

(3) $Q_2 * R = [p * Q_0 + (1-p) * N_0(Q)] * R = p * Q_0 * R + (1-p) * N_0(Q) * R$

(4) $\forall\ i$, $1 \le i \le n$, $\forall\ j$, $1 \le j \le m$,

$$Q_{3i,j} = P * \sum_{k=1}^{m+1} Q_{0i,k} * R_{k,j} + (1-p) * \sum_{k=1}^{m+1} N_0(Q)_{i,k} * R_{k,j}.$$

(5) We use: $Q_{0i,n+1} = 0$, $R_{k,j} = 0$, $1 \le k \le m$, $k \ne j$, $R_{k,j} = 1$, $1 \le k \le m$, $k = j$.

(6) Thus, $Q_{3i,j} = p * Q_{0i,j} + (1-p) * \sum_{k=1} m + 1 N_0(Q)_{i,k} * R_{k,j}$.

(7) We use $Q_{0i,j} = Q_{i,j}$, $1 \leq i \leq n$

(8) and

$$\begin{array}{ll} N_0(Q)_{i,j} = 0, & 1 \leq j \leq m,\ 1 \leq i \leq n \\ N_0(Q)_{i,m+1} = 1, & 1 \leq i \leq n \\ R_{m+1,j} = m, & 1 \leq j \leq m \end{array}$$

(9) Thus, $Q_{3i,j} = p * Q_{i,j} + (1-p) * 1 * m_j = p * Q_{i,j} + (1-p) * m_j$.

That is, for each term $\forall\ i$, $1 \leq i \leq n$, $\forall\ j$, $1 \leq j \leq m$.

(10) $Q_{3i,j} = Q_{1i,j}$.

(11) Thus, $Q_3 = Q_1 = p * Q + (1-p) * M(Q)$

(12) Hence, $Q_2 * R = Q_1$. According to the 2nd condition of Blackwell's Theorem, Q_2 is generally more informative than Q_1. □

Proof of Theorem 1.

(1) Denote $Q_3 = p * Q + (1-p) * M(Q)$. According to Lemma 1 of this theorem (Theorem 1), Q_2 is generally more informative than Q_3.

(2) Denote $Q_4 = p * Q_0 + (1-p) * M_0(Q)$. That is, Q_3 with the addition of a no-information signal. Q_3 and Q_4 have the same level of general informativeness [1]. Thus from the transitivity of the general informativeness ratio it is clear that Q_2 is generally more informative than Q_4.

(3) $Q_1 = p * Q_0 + p_1 * M_0(Q) + p_2 * N_0(Q)$, $0 \leq p_1$, $0 \leq p_2$, $p_1 + p_2 = 1 - p$

(4) $Q_1 = \dfrac{p_1 + p_2}{p_1 + p_2} * (p * Q_0 + p_1 * M_0(Q) + p_2 * N_0(Q))$

(5) $Q_1 = \dfrac{p_1}{p_1 + p_2} * (p * Q_0 + (p_1 + p_2) * M_0(Q)) + \dfrac{p_2}{p_1 + p_2} * (p * Q_0 + (p_1 + p_2) * N_0(Q))$

(6) Using $p_1 + p_2 = 1 - p$,

$$Q_1 = \frac{p_1}{p_1 + p_2} * (p * Q_0 + (1-p) * M_0(Q)) + \frac{p_2}{p_1 + p_2} * (p * Q_0 + (1-p) * N_0(Q))$$

(7) Using $Q_2 = p * Q_0 + (1-p) * N_0(Q)$ and in (2) $Q_4 = p * Q_0 + (1-p) * M_0(Q)$, then

$$Q_1 = \frac{p_1}{p_1 + p_2} * Q_2 + \frac{p_2}{p_1 + p_2} * Q_4$$

(8) Thus, (2) $\Rightarrow Q_2$ is generally more informative than Q_4. Hence, from Theorem 1 in [5], it is concluded that Q_2 is generally more informative than Q_1, which is a convex combination of Q_2 and Q_4. $\square$

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Received April 23, 2009
Final Accepted June 1, 2009