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SOLVING RATIO-DEPENDENT PREDATOR-PREY SYSTEM WITH CONSTANT EFFORT HARVESTING USING VARIATIONAL ITERATION METHOD

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Abstract. Due to wide range of interest in use of bio-economic models to gain insight in to the scientific management of renewable resources like fisheries and forestry, variational iteration method (VIM) is employed to approximate the solution of the ratio-dependent predator-prey system with constant effort prey harvesting. The results are compared with the results obtained by Adomian decomposition method and reveal that VIM is very effective and convenient for solving nonlinear differential equations.

1. Introduction. From the point of view of human needs, the exploitation of biological resources and guaranteed continuous harvesting of populations in fishery, forestry, and wildlife management are of great importance. There is a wide range of interest in the use of bio-economic models to gain insight in to the scientific management of renewable resources like fisheries and forestries.

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This is related to the optimal management of renewable resources [1]. Generally speaking, it is necessary to investigate the sustainability of harvesting of populations in some models. Taking in to consideration the above reasons, we focus on the ratio-dependent predator-prey model with constant effort harvesting [2, 3, 4]. The reason for the model is that numerous field and laboratory experiments and observations showed that functional and numerical responses over typical ecological timescales ought to depend on the densities of both prey and predators, especially when predators must search for food and therefore share or compete for food [5].

The suitable functional response is a ratio-dependent response function in which the per capita predator growth rate should be a function of the ratio of prey to predator abundance.

In this paper, we assume that the predator in the model is not of commercial importance. The prey is subjected to constant effort harvesting with a parameter measuring the effort being spent by a harvesting agency. The harvesting activity does not affect the predator Population directly. It is obvious that the harvesting activity does reduce the predator population indirectly by reducing the availability of the prey to the predator. Adopting a simple logistic growth for prey population and $e > 0$, $b > 0$, $c > 0$ representing the predator death rate, capturing rate and conversion rate, respectively, we formulate the problem as:

$$\frac{dx}{dt} = x(1-x) - \frac{bxy}{y+x} - rx, \tag{1}$$

$$\frac{dy}{dt} = \frac{cxy}{y+x} - ey, \tag{2}$$

Where $x(t)$ and $y(t)$ represent the fractions of population densities for prey and predator at time t, respectively. Eqs. (1) and (2) are to be solved subject to the biologically meaningful initial conditions $x(0) \geq 0$ and $y(0) \geq 0$. A qualitative investigation of the system described by Eqs. (1) and (2) reveals that the long-term behaviour, falls in to three categories: mutual extinction, predator extinction and coexistence[6].When both prey and predator go extinct for some values of parameters, the solution asymptotically approaches equilibrium E_0 of the form:

$$E_0 = (0, 0). \tag{3}$$

The eigenvalues of the jacobian matrix evaluated at the E_0 reveal that the mutual extinction equilibrium is a local asymptotically stable node provided

$r + b > 1$, regardless of their initial densities. This clearly shows that over-exploitation of the prey population by constant effort harvesting process together with high predator capturing rate can lead to mutual extinction. When only the predator population become extinct, the solution asymptotically approaches equilibrium E_1 of the form:

$$E_1 = (1 - r, 0). \tag{4}$$

The eigenvalues of the jacobian matrix evaluated at the equilibrium E_1 shows that the predator extinction equilibrium is a local asymptotically stable node provided the predator death is greater than conversion rate, that is, $c < e$, another long-term possibility is the predator-prey coexistence equilibrium E_2 of the form:

$$E_2 = \left(1 - r - (c - e)\frac{b}{c}, \frac{(1 - r)c - b(c - e))(c - e)}{ce}\right). \tag{5}$$

The eigenvalue δ of the jacobian matrix evaluated at E_2 satisfies:

$$\delta^2 - \frac{((r + b - e - 1)c^2 + (c - b)e^2)\delta}{c^2} - \frac{e(c - e)(cr + bc - be - c)}{c^2} = 0. \tag{6}$$

Hence, E_2 is locally asymptotically stable provided:

$$\frac{b(c - e)}{c(1 - r)} < 1 \quad \text{and} \quad \frac{c^2(r + b - 1) - be^2}{ec(c - e)} < 1 \tag{7}$$

From Eq. (5), we observe that whenever coexistence equilibrium E_2 occurs, the predator extinction equilibrium E_1 becomes an unstable saddle point, since predator death rate must be less than the conversion rate, that is, $c > e$.

In this paper we find analytical approximate of system (1-2) using VIM [7]–[16]. The accuracy of the solutions is demonstrated through some numerical examples. Four cases are discussed in details and the results are compared with those found by Adomian decomposition method (ADM) [23].

Over the last decades several analytical/approximate methods have been developed to solve ordinary and partial differential equations. Some of these techniques include homotopy perturbation method (HPM) [17]–[22], variational iteration method (VIM) [7]–[16], etc.

He [14]–[16] proposed a variational iteration method based on the use of restricted variations and correction functionals which has found a wide application for the solution of nonlinear ordinary and partial differential equations.

This method does not require the presence of small parameters in the differential equation, and provides the solution (or an approximation to it) as a sequence of iterates. The method does not require that the nonlinearities be differentiable with respect to the dependent variable and its derivatives.

2. The variational iteration method. To clarify the basic ideas of VIM, we consider the following differential equation:

$$Lu + Nu = g(t), \tag{8}$$

where L is a linear operator, N is a nonlinear operator and $g(t)$ is a homogeneous term.

According to VIM, we can write down a correction functional as follows:

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda\left(Lu_n(\tau) + N\tilde{u}_n(\tau) - g(\tau)\right) d\tau, \tag{9}$$

where λ is a general lagrangian multiplier which can be identified optimally via the variational theory. The subscript n indicates the nth approximation and u_n is considered as a restricted variation, i.e., $\delta\tilde{u}_n = 0$.

3. Applications. To solve the system (1–2) by means of VIM, at first, we calculate the common denominator and multiply both sides of the equations by the obtained common dominator, in order to obtain the Eq.(10) as:

$$\begin{aligned} \frac{dx}{dt} &= \frac{xy - bxy - rxy - yx^2 + x^2 - rx^2 - x^3}{(x+y)} \\ &= \frac{-yx^2 - x^3 + x^2(1-r) + xy(1-b-r)}{(x+y)} \\ \frac{dy}{dt} &= \frac{cxy - exy - ey^2}{(x+y)} \end{aligned} \tag{10}$$

one can construct the following correction functional,

$$x_{n+1}(t) = x_n(t) + \int_0^t \lambda_1 \left(\frac{y_n(\tau)\left(\frac{d}{d\tau}x_n(\tau)\right) + x_n(\tau)\left(\frac{d}{d\tau}x_n(\tau)\right) - (1-b-r)x_n(\tau)}{y_n(\tau) + y_n(\tau)x_n^2(\tau) - (1-r)x_n^2(\tau) + x_n^3(\tau)} \right) d\tau \tag{11}$$

$$y_{n+1}(t) = y_n(t) + \int_0^t \lambda_2 \left(\begin{array}{l} y_n(\tau)\left(\frac{d}{d\tau}y_n(\tau)\right) + x_n(\tau)\left(\frac{d}{d\tau}y_n(\tau)\right) - cx_n(\tau) \\ y_n(\tau) + ey_n^2(\tau) + ex_n(\tau)y_n(\tau) \end{array} \right) d\tau \tag{12}$$

The following stationary conditions can be obtained:

$$\begin{aligned} \lambda_1' \,|_{\tau=t} &= 0\,, \\ 1 + \lambda_1 \,|_{\tau=t} &= 0\,, \\ \lambda_2' \,|_{\tau=t} &= 0\,, \\ 1 + \lambda_2 \,|_{\tau=t} &= 0\,. \end{aligned} \tag{13}$$

We obtain the lagrangian multipliers:

$$\begin{aligned} \lambda_1 &= -1 \\ \lambda_2 &= -1. \end{aligned} \tag{14}$$

Substituting the values of λ_1 and λ_2 from Eq. (14) in to correction functional of Eqs. (11) and (12) leads to the following iteration formulae:

$$\begin{aligned} x_{n+1}(t) &= x_n(t) \\ &- \int_0^t \left(\begin{array}{l} y_n(\tau)\left(\frac{d}{d\tau}x_n(\tau)\right) + x_n(\tau)\left(\frac{d}{d\tau}x_n(\tau)\right) - (1-b-r)x_n(\tau) \\ y_n(\tau) + y_n(\tau)x_n^2(\tau) - (1-r)x_n^2(\tau) + x_n^3(\tau) \end{array} \right) d\tau \\ y_{n+1}(t) &= y_n(t) \\ &- \int_0^t \left(\begin{array}{l} y_n(\tau)\left(\frac{d}{d\tau}y_n(\tau)\right) + x_n(\tau)\left(\frac{d}{d\tau}y_n(\tau)\right) - cx_n(\tau) \\ y_n(\tau) + ey_n^2(\tau) + ex_n(\tau)y_n(\tau) \end{array} \right) d\tau \end{aligned} \tag{15}$$

3.1. Case 1. Parameter values used for case 1 are shown in Table 1.

Table 1. Parameter values used for case 1

Case	x_0	y_0	b	c	e	r	comments
1	0.5	0.3	0.8	0.2	0.5	0.9	E_0 (stable-mutual extinction)

Now we start with an arbitrary initial approximations that satisfy the initial conditions

$$\begin{aligned} x_0(t) &= x(0) = 0.5, \\ y_0(t) &= y(0) = 0.3. \end{aligned} \tag{16}$$

Using the initial guess by Eq. (16) and by the iteration formula (15), one can obtain the following results:

$$x_1(t) = 0.5 - 0.28\,t \tag{17}$$

$$y_1(t) = 0.3 - 0.09t \tag{18}$$

$$x_2(t) = 0.5 - 0.336t + 0.1376t^2 - 0.0587t^3 + 0.007252t^4 + \cdots \tag{19}$$

$$y_2(t) = 0.3 - 0.108t + 0.0162t^2 - 0.00387t^3 \tag{20}$$

$$\begin{aligned} x_3(t) = {} & 0.5 - 0.3472t + 0.180208t^2 - 0.09476t^3 + 0.044127t^4 \\ & -0.02464t^5 + 0.011555t^6 - 0.004523t^7 + 0.001407t^8 \\ & -0.000338t^9 + 0.0000647t^{10} + \cdots \end{aligned} \tag{21}$$

$$\begin{aligned} y_3(t) = {} & 0.3 - 0.1116t + 0.018684t^2 - 0.0025728t^3 - 0.000507t^4 \\ & +0.0000867t^5 - 0.0000364t^6 - 3.8136\times10^{-6}t^7 \\ & +1.0524\times10^{-6}t^8 \end{aligned} \tag{22}$$

$$\begin{aligned} x_4(t) = {} & 0.5 - 0.34944t + 0.19124t^2 - 0.104500t^3 + 0.056028t^4 \\ & -0.032292t^5 + 0.017096t^6 - 0.00933t^7 + 0.00536t^8 \\ & -0.00320t^9 + 0.001882t^{10} + \cdots \end{aligned} \tag{23}$$

$$\begin{aligned} y_4(t) = {} & 0.3 - 0.11232t + 0.01886t^2 - 0.00156t^3 - 0.000622t^4 \\ & +0.000108t^5 - 0.000049t^6 + 4.7822\times10^{-6}t^7 - 2.94371\times10^{-6}t^8 \\ & -1.8778\times10^{-6}t^9 + 2.0335\times10^{-6}t^{10} + \cdots \end{aligned} \tag{24}$$

$$(25)\quad x_5(t) = 0.5 - 0.34988t + 0.19394t^2 - 0.10671t^3 + 0.05903t^4 - 0.03367t^5 + 0.01848t^6 - 0.01057t^7 + 0.00608t^8 - 0.003441t^9 + 0.00187t^{10} + \cdots$$

$$(26)\quad y_5(t) = 0.3 - 0.11246t + 0.0188383t^2 - 0.00123181t^3 - 0.000463283t^4 + 0.0000568033t^5 - 0.00002803t^6 - 2.95789t^7 + 1.27769t^8 - 2.011\times 10^{-6}t^9 - 4.68418\times 10^{-7}t^{10}$$

And so on. In the same way the rest of the components of the iteration formula can be obtained.

3.2. Case 2. Parameter values used for case 2 are shown in Table 2.

Table 2. Parameter values used for case 2

Case	x_0	y_0	b	c	e	r	comments
2	0.5	0.3	0.8	0.2	0.5	0.1	E_1 (stable-predator extinction)

Now we start with an arbitrary initial approximations that satisfy the initial conditions:

$$(27)\quad \begin{aligned} x_0(t) &= x(0) = 0.5 \\ y_0(t) &= y(0) = 0.3 \end{aligned}$$

Using the initial guess by Eq. (27) and by the iteration formula (15), one can obtain the following results,

$$(28)\quad x_1(t) = 0.5 + 0.04t$$

$$(29)\quad y_1(t) = 0.3 - 0.09t$$

$$(30)\quad x_2(t) = 0.5 + 0.048t + 0.0184t^2 + 0.0011t^3 + 0.000032t^4$$

$$(31)\quad y_2(t) = 0.3 - 0.108t + 0.0162t^2 - 0.00099t^3$$

$$(32)\quad x_3(t) = 0.5 + 0.0496t + 0.01304t^2 - 0.0005416t^3 - 0.000337t^4 - 0.000122t^5 - 0.000013t^6 - 2.1352\times 10^{-6}t^7 - 1.96771\times 10^{-7}t^8 - 1.14827\times 10^{-8}t^9 - 4.275324\times 10^{-10}t^{10} + \cdots$$

$$(33)\quad y_3(t) = 0.3 - 0.1116t + 0.02214t^2 - 0.002712t^3 + 0.00029308t^4 - 0.000035t^5 + 2.73465 \times 10^{-6}t^6 - 3.027857\times 10^{-8}t^7 + 1.188\times 10^{-9}t^8$$

$$
\begin{aligned}
(34)\ x_4(t) = {} & 0.5 + 0.04992t + 0.0123296t^2 - 0.001226t^3 - 0.0003025t^4 \\
& -0.0000768t^5 + 4.6\times10^{-6}t^6 + 3.14894\times10^{-6}t^7 + 6.9\times10^{-7}t^8 \\
& +6.649141\times10^{-8}t^9 - 2.3057\times10^{-9}t^{10}
\end{aligned}
$$

$$
\begin{aligned}
(35)\ y_4(t) = {} & 0.3 - 0.11232t + 0.0238464t^2 - 0.003553t^3 + 0.000414t^4 \\
& -0.00005t^5 + 4.5328\times10^{-6}t^6 - 8.21347\times10^{-8}t^7 \\
& +1.94445\times10^{-8}t^8 - 3.11349\times10^{-9}t^9 - 2.2\times10^{-10}t^{10} + \cdots
\end{aligned}
$$

$$
\begin{aligned}
(36)\ x_5(t) = {} & 0.5 + 0.049984t + 0.01226t^2 - 0.0014827t^3 - 0.00026116t^4 \\
& -0.000044497t^5 + 0.00001179t^6 + 3.054\times10^{-6}t^7 \\
& +3.51522\times10^{-7}t^8 - 9.3075\times10^{-8}t^9 - 2.502\times10^{-8}t^{10} + \cdots
\end{aligned}
$$

$$
\begin{aligned}
(37)\ y_5(t) = {} & 0.3 - 0.112464t + 0.02429049t^2 - 0.0038529t^3 + 0.000462493t^4 \\
& -0.000047929t^5 + 4.2648\times10^{-6}t^6 + 9.71457\times10^{-9}t^7 \\
& -2.94208\times10^{-8}t^8 - 3.7068\times10^{-9}t^9 - 5.92532\times10^{-10}t^{10} + \cdots
\end{aligned}
$$

And so on. In the same way the rest of the components of the iteration formula can be obtained.

3.3. Case 3. Parameter values used for case 3 are shown in Table 3.

Table 3. Parameter values used for case3

Case	x_0	y_0	b	c	e	r	comments
3	0.3	0.6	0.5	0.5	0.3	0.1	E_2 (stable-coexistence)

Now we start with an arbitrary initial approximations that satisfy the initial conditions:

$$
\begin{aligned}
x_0(t) &= x(0) = 0.3 \\
y_0(t) &= y(0) = 0.6
\end{aligned}
\tag{38}
$$

Using the initial guess by Eq. (38) and by the iteration formula (15), one can obtain the following results,

$$(39)\quad x_1(t) = 0.3 + 0.072t$$

$$(40)\quad y_1(t) = 0.6 - 0.072t$$

$$(41)\ x_2(t) = 0.3 + 0.0792t + 0.00432t^2 - 0.000691t^3$$

$$(42)\ y_2(t) = 0.6 - 0.0792t + 0.01512t^2 - 0.000864t^3$$

$$\begin{aligned}(43)\ x_3(t) = {} & 0.3 + 0.07992t + 0.005184t^2 - 0.001051484t^3 - 0.000189t^4 \\ & -4.8049 \times 10^{-6}t^5 - 5.8724352 \times 10^{-8}t^6 + 3.46246 \times 10^{-7}t^7 \\ & -3.14424 \times 10^{-9}t^8 - 2.06391 \times 10^{-9}t^9 + 7.430082 \times 10^{-11}t^{10}\end{aligned}$$

$$\begin{aligned}(44)\ y_3(t) = {} & 0.6 - 0.07992t + 0.018144t^2 - 0.001957824t^3 + 0.0000886t^4 \\ & -3.79468\times10^{-7}t^5 + 1.617408\times10^{-7}t^6 - 1.492992\times10^{-8}t^7\end{aligned}$$

$$\begin{aligned}(45)\ x_4(t) = {} & 0.3 + 0.079992t + 0.0053136t^2 - 0.0011375t^3 - 0.00024943t^4 \\ & +3.222\times10^{-6}t^5 + 5.79944\times10^{-6}t^6 + 8.23651\times10^{-7}t^7 \\ & +3.394408\times10^{-8}t^8 - 2.429502\times10^{-8}t^9 \\ & -1.52498\times10^{-9}t^{10} + \cdots\end{aligned}$$

$$\begin{aligned}(46)\ y_4(t) = {} & 0.6 - 0.079992t + 0.018597t^2 - 0.0022458t^3 + 0.0001217t^4 \\ & +8.06091\times10^{-8}t^5 + 1.78094\times10^{-7}t^6 - 2.274363\times10^{-7}t^7 \\ & +2.5336\times10^{-8}t^8 - 1.94796\times10^{-9}t^9 + 2.965963\times10^{-10}t^{10} + \cdots\end{aligned}$$

$$\begin{aligned}(47)\ x_5(t) = {} & 0.3 + 0.0799992t + 0.00533t^2 - 0.001152834336t^3 \\ & -0.000260956t^4 + 6.5696\times10^{-6}t^5 + 8.4282\times10^{-6}t^6 \\ & +7.07789\times10^{-7}t^7 - 1.2185\times10^{-7}t^8 \\ & -4.27059\times10^{-8}t^9 - 3.25465\times10^{-9}t^{10} + \cdots\end{aligned}$$

$$\begin{aligned}(48)\ y_5(t) = {} & 0.6 - 0.0796224t + 0.01844883t^2 - 0.002342233t^3 \\ & +0.00016118917t^4 - 1.1576\times10^{-6}t^5 - 1.046\times10^{-6}t^6 \\ & -2.5511\times10^{-7}t^7 + 4.66417\times10^{-8}t^8 - 4.41784\times10^{-9}t^9 \\ & -7.2688\times10^{-10}t^{10} + \cdots\end{aligned}$$

And so on. In the same way the rest of the components of the iteration formula can be obtained.

3.4. Case 4. Parameter values used for case 4 are shown in Table 4.

Table 4: Parameter values used for case4

Case	x_0	y_0	b	c	e	r	comments
4	0.5	0.2	0.5	0.5	0.1	0.2	E_2(stable-coexistence)

Now we start with an arbitrary initial approximations that satisfy the initial conditions

$$x_0(t) = x(0) = 0.5 \qquad (49)$$
$$y_0(t) = y(0) = 0.2$$

Using the initial guess by Eq. (49) and by the iteration formula (15), one can obtain the following results,

$$(50)\quad x_1(t) = 0.5 + 0.055t$$

$$(51)\quad y_1(t) = 0.2 + 0.036t$$

$$(52)\quad x_2(t) = 0.5 + 0.0715t - 0.0067775t^2 - 0.0013695t^3 - 0.00006881t^4$$

$$(53)\quad y_2(t) = 0.2 + 0.0468t + 0.003442t^2 + 0.00022t^3$$

$$\begin{aligned}(54)\quad x_3(t) = {} & 0.5 + 0.07645t - 0.011819975t^2 - 0.002022698417t^3 \\ & +0.00021469t^4 + 0.000066233t^5 + 2.092397\times10^{-6}t^6 \\ & -7.203366\times10^{-7}t^7 - 8.99976\times10^{-8}t^8 - 1.94341\times10^{-9}t^9 \\ & +3.89469\times10^{-10}t^{10} + \cdots\end{aligned}$$

$$\begin{aligned}(55)\quad y_3(t) = {} & 0.2 + 0.05004t + 0.00486838t^2 + 0.0002228067333t^3 \\ & -0.0000341309t^4 - 4.8145\times10^{-6}t^5 - 4.48292\times10^{-7}t^6 \\ & -2.4999\times10^{-8}t^7 - 7.59759\times10^{-10}t^8 + \cdots\end{aligned}$$

$$\begin{aligned}(56)\quad x_4(t) = {} & 0.5 + 0.077935t - 0.0143233t^2 - 0.0018866t^3 + 0.0005497452t^4 \\ & +0.000079463t^5 - 0.00001651t^6 - 3.3788\times10^{-6}t^7 \\ & +2.573138193\times10^{-7}t^8 + 1.026083\times10^{-7}t^9 \\ & -2.009319\times10^{-10}t^{10} + \cdots\end{aligned}$$

$$\begin{aligned}(57)\quad y_4(t) = {} & 0.2 + 0.051t + 0.0053569t^2 + 0.000143315t^3 \\ & -0.000057589t^4 - 5.27847\times10^{-6}t^5 + 9.580461\times10^{-8}t^6 \\ & +1.176407808\times10^{-7}t^7 + 1.7363125\times10^{-8}t^8 \\ & +1.053836\times10^{-9}t^9 - 7.0356395\times10^{-11}t^{10} + \cdots\end{aligned}$$

$$x_5(t) = 0.5 + 0.078380t - 0.015379441t^2 - 0.00160042077t^3 + 0.000717328t^4 + 0.0000377t^5 - 0.00003352117t^6 - 0.0000014463t^7 + 0.00000136764t^8 + 7.92469 \times 10^{-8}t^9 - 4.96455\times10^{-8}t^{10} + \cdots \tag{58}$$

$$y_5(t) = 0.2 + 0.0513036t + 0.005516518t^2 + 0.000082204t^3 - 0.0000618682t^4 - 3.0147\times10^{-6}t^5 + 6.179182\times10^{-7}t^6 + 1.47148\times10^{-7}t^7 + 1.8342956 \times 10^{-9}t^8 - 3.564454\times10^{-9}t^9 - 5.81218\times10^{-10}t^{10} + \cdots \tag{59}$$

And so on. In the same way the rest of the components of the iteration formula can be obtained.

4. Numerical results. For comparison with the results done by ADM [23], some numerical results of $x(t)$, $y(t)$ VIM and ADM [23] are presented in Tables.(5–8).

Table 5. Comparison between results of VIM and ADM for case 1

t	X_{ADM}	X_{VIM}	Y_{ADM}	Y_{VIM}
0.1	0.466845	0.466849	0.288936	0.288940
0.2	0.437017	0.437011	0.278242	0.278250
0.3	0.410035	0.410015	0.267909	0.267919
0.4	0.385485	0.385474	0.257929	0.257382
0.5	0.362972	0.363064	0.248294	0.248295

Table 6. Comparison between results of VIM and ADM for case 2

t	X_{ADM}	X_{VIM}	Y_{ADM}	Y_{VIM}
0.1	0.505121	0.505119	0.288990	0.288992
0.2	0.510477	0.510474	0.278446	0.278448
0.3	0.516058	0.516056	0.268345	0.268346
0.4	0.521853	0.521853	0.258666	0.258665
0.5	0.527850	0.527854	0.249389	0.249386

Table 7. Comparison between results of VIM and ADM for case 3

t	X_{ADM}	X_{VIM}	Y_{ADM}	Y_{VIM}
0.1	0.308052	0.308052	0.592184	0.592219
0.2	0.316203	0.316203	0.584728	0.584794
0.3	0.324446	0.324446	0.577618	0.577711
0.4	0.332772	0.332777	0.570842	0.570957
0.5	0.341172	0.341172	0.564386	0.564518

Table 8. Comparison between results of VIM and ADM for case 4

t	X_{ADM}	X_{VIM}	Y_{ADM}	Y_{VIM}
0.1	0.507695	0.507682	0.205198	0.205185
0.2	0.515064	0.5150492	0.210509	0.210481
0.3	0.522102	0.522092	0.215932	0.215889
0.4	0.528806	0.528807	0.221466	0.221407
0.5	0.535174	0.535190	0.227112	0.227037

5. Conclusion. Variational iteration method is employed to approximate the solution of the ratio-dependent predator-prey system with constant effort prey harvesting. The results obtained here were compared with results of Adomian decomposition method. There is less computations involved in the proposed method as compared to ADM.

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