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STUDY OF QUEUING SYSTEMS WITH A GENERALIZED DEPARTURE PROCESS[*]

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Abstract. This paper deals with a full accessibility loss system and a single server delay system with a Poisson arrival process and state dependent exponentially distributed service time. We use the generalized service flow with nonlinear state dependence mean service time. The idea is based on the analytical continuation of the Binomial distribution and the classic M/M/n/0 and M/M/1/k system. We apply techniques based on birth and death processes and state-dependent service rates.

We consider the system M/M(g)/n/0 and M/M(g)/1/k (in Kendal notation) with a generalized departure process Mg. The output intensity depends nonlinearly on the system state with a defined parameter: "*peaked factor p*". We obtain the state probabilities of the system using the general solution of the birth and death processes.

The influence of the peaked factor on the state probability distribution, the congestion probability and the mean system time are studied. It is shown that the state-dependent service rates changes significantly the characteristics of the queueing systems. The advantages of simplicity and uniformity in representing both peaked and smooth behaviour make this queue attractive in network analysis and synthesis.

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Key words: Loss and delay system, Queueing analyses, State dependent service rate, Birth and death process, Peaked and smooth traffic.

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1. Introduction.

Simple models like the classical full accessibility and single-server queues can often be used to obtain comprehensive results, e.g., to predict the global traffic behaviour. When modeling network traffic, packet and connection arrivals are often assumed to be Poisson processes because such processes have attractive theoretical properties.

Many studies on traffic measurements from a variety of packet switching networks, like Ethernet, Internet, ATM, etc., have shown considerable difference between actual network traffic and assumptions in traditional theoretical traffic models. The basic characteristic of traffic in modern telecommunications networks is burstness. That is why there are many studies that generalize the queuing systems by state-dependent arrival and service rates.

In [5] the burstness of the total arrival process is characterized in packet network performance models by the dependence among successive interarrival times, dependence among successive service times and between service and interarrival times. These dependence effects are demonstrated analytically by considering a multiclass single-server queue with batch-Poisson arrival processes.

In [7] the author has modified the generalized Erlang blocking model to permit blocked requests to retry, with reduced resource requirements and arbitrary mean residency requirements. The presented approach modifies a one-dimensional recursion developed for the generalized Erlang model in an intuitively satisfying manner, and results in an approximation scheme that is both efficient and quite accurate. This study arose in the context of high-speed networks in which high bandwidth but non-real-time messages may, upon being blocked, request service with smaller bandwidth and larger residency time.

[9] is focused on the calculation of call blocking probabilities in single link loss models where calls of each service-class come from finite sources and compete for the available link bandwidth under the complete sharing policy. The Engset multirate and single-retry loss models for finite sources are there reviewed where blocked calls of a service-class may immediately retry once in order to be connected within the system with reduced bandwidth and increased service time requirements.

Two generalizations of the Engset model are considered [11], which permit: (i) different holding and interarrival time distributions from source to source; (ii) different time distribution until a source generates a new burst or packet depending on whether the previous burst or packet was successful or not. Call and time congestions are approximated for the generalization. The approximation accuracy is validated and an efficient algorithm for numerical computation are suggested and its convergence is proved.

In [4] an algorithm is developed for computing exact steady-state blocking probabilities for each class in product-form loss networks to cover general state-dependent arrival and service rates. This generalization allows considering a wide variety of buffered and unbuffered resource-sharing models with non-Poisson traffic as may arise with overflows in the context of alternative routing.

In [10] a numerically exact method is developed for evaluating the time-dependent mean, variance, and higher order moments of the number of entities in a $Pht/Pht/\infty$ queueing system, where Pht denotes a time-dependent generalization of a phase-type renewal process.

In [1] a continuous-time M/M/1 queueing system is analyzed in which the server can serve at two different speeds. The actual speed of the server depends on the state (empty or nonempty) of a fluid buffer. Fluid flows continuously into the fluid buffer at a constant rate, but is released from the buffer only during busy periods of the server. Hence, the contents of the fluid buffer are in turn determined by the queueing system. The queueing model serves as a mathematical model for a two-level *traffic shaper* at the edge of an ATM network. The stationary joint distribution of the number of customers in the system and the contents of the fluid buffer is investigated. From this distribution, various performance measures such as the steady-state sojourn time distribution of a customer is obtained.

In [8] a generalized Poisson arrival process by state-dependent arrival rates is introduced and evaluated. The proposed single server delay system provides a unified framework to model peaked and smooth traffic and makes it attractive in network analysis.

In [3] a queueing system is presented where feedback information about the level of congestion is given right after arrival instants. If the amount of work right after an arrival is smaller or larger than a finite number then the server starts to work at two different service speeds. In addition, the authors have considered the generalization to the N-step service speed function.

In [2] a TCP-like linear-increase multiplicative-decrease flow control mechanism is presented. The authors consider congestion signals that arrive in batches according to a Poisson process. The service times in the queuing model depend on the workload in the system and the transmission rate cannot exceed a certain maximum value.

The Bernoulli-Poisson-Pascal (BPP) method is used to approximate the main congestion functions associated with peaked and smooth traffic in lost-call-cleared systems. The BPP model represents peaked and smooth traffic by two separate models, and cannot represent arbitrary smooth traffic. The BPP traffic models are insensitive to the holding time distribution [4]. The state probabilities for these loss systems only depend on the holding time through the mean value which is included in the offered traffic.

The literature of queuing contains many studies about queues with workload-dependent service speeds. In these studies it is usually assumed that the speed of the server is continuously adapted over time based on the buffer content. In many practical situations service speed adaptations are only made at particular points in time, like arrival epochs. For example, feedback information about the buffer state may only be available at such epochs.

In this paper, we consider queueing systems with adaptable service speed based on the amount of work right after customer arrivals or departure. Between these events, the service speed is held fixed and may not be changed until the next customer arrival or depart. We generalize the classical loss and delay queueing systems to nonlinear state-dependent service rate. We use the generalized service flow with nonlinear state dependence mean service time. The idea is based on the analytic continuation of the binomial distribution and the classic M/M/n/0 and M/M/1/k system. We apply techniques based on birth and death processes and state-dependent service rates.

These generalized models can be used to analyze multiplexing, message storage, traffic regulator and communication network performance.

2. Generalized erlang distribution.

Let us consider a full availability loss system M/M(g)/S/0/S with a Poisson input stream M, state dependent exponentially distributed service time M(g), number of servers S, waiting room 0 and number of sources S. This is a birth and death process and we can use the general solution for the stationary probability of having j customers in the system [4]:

$$P_j = \frac{\prod_{i=0}^{j-1} \lambda_i/\mu_{i+1}}{1 + \sum_{v=1}^{S} \prod_{i=0}^{v-1} \lambda_i/\mu_{i+1}} \qquad j = 1, 2, 3, \dots, S \ . \tag{1}$$

This generalized queueing system may be described by selecting the birth and death coefficient as follows:

$$\lambda_j = \lambda, \quad \mu_j = j\,\mu j^{1-p} \quad j = 0, 1, 2, \dots, S. \tag{2}$$

The service rate is state-dependent and depends on the peakedness factor p. This system is always ergodic. The finite state-transition diagram is shown in Fig. 1.

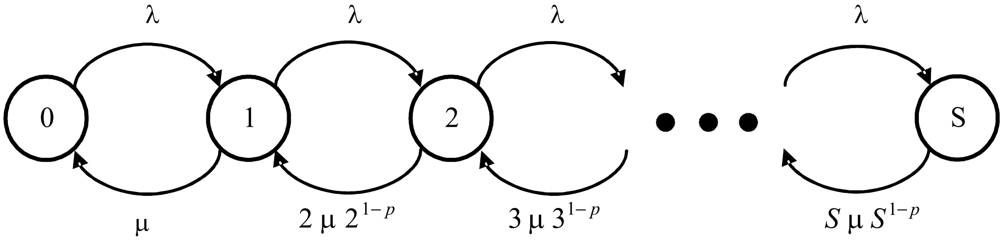


Fig. 1. A state-transition diagram – M/M(g)/S/0/S system

As the number of servers is equal to the number of sources the system has no losses and delay, the whole offered traffic is carried and it is called the intended traffic load.

The stationary probabilities of having j customers in the system has a generalized Erlang distribution when the service time is state dependent

$$P_j = \frac{a^j \big/ (j!)^{2-p}}{\sum_{i=0}^{S} a^i \big/ (i!)^{2-p}} \quad j = 0, 1, 2, \ldots, S, \tag{3}$$

where $a = \lambda/\mu$ is traffic intensity.

The intended traffic is the equilibrium number of busy servers

$$A_i = \sum_{j=1}^{S} j\, P_j. \tag{4}$$

The variance of the intended traffic is

$$V(A_i) = \sum_{j=0}^{S} (j - A_i)^2 P_j. \tag{5}$$

The peakedness of the intended traffic is the variance to mean ratio

$$z_i = V(A_i)/A_i. \tag{6}$$

3. Model description.

Generalized full accessibility loss system. Let us consider a multi-server system M/M(g)/n/0/S with a Poisson input stream M, state dependent exponentially distributed service time M(g), number of servers n, waiting room 0 and number of sources S (S > n). This generalized loss system may be described by selecting the birth-death coefficient as follows

$$(7) \qquad \lambda_j = \lambda \qquad \mu_j = j\,\mu\, j^{1-p} \qquad j = 0, 1, 2, \ldots, n.$$

The finite state-transition diagram is shown in Fig.2.

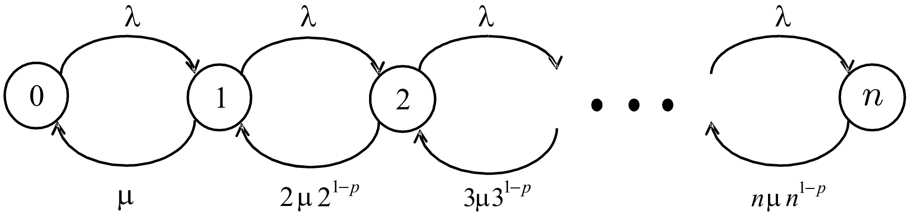


Fig. 2. A state-transition diagram – M/M(g)/n/0/S system

Applying these coefficients to the general solution of the birth and death process and using traffic intensity $a = \lambda/\mu$ we obtain the steady state probabilities

$$(8) \qquad P'_j = \frac{a^j \Big/ (j!)^{2-p}}{\sum_{i=0}^{n} a^i \big/ (i!)^{2-p}} \qquad j = 0, 1, 2, \ldots, n.$$

The offered traffic is calculated by means of the arrival rate and the mean holding time

$$(9) \qquad M_k = -z_R.\lambda_1 \left(F^2_{\delta_N} - F^2_{\delta_S}\right) \sin\Theta.$$

The carried traffic is equivalent to the average number of busy servers

$$(10) \qquad M_k = -z_R.\lambda_1 \left(F^2_{\delta_N} - F^2_{\delta_S}\right) \sin\Theta.$$

Generalized single server delay system. Let us consider a single server queue M/M(g)/1/k with a Poisson input stream M, state dependent exponentially distributed service time M(g) and limited waiting rooms k. This generalized queueing system has birth and death coefficient as follows

$$\lambda_j = \lambda \qquad \mu_j = \mu\, j^{1-p} \qquad j = 0, 1, 2, \ldots, k+1. \tag{11}$$

The finite state-transition diagram is shown in Fig. 3.

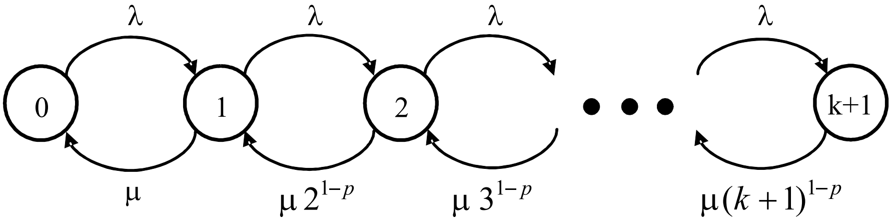


Fig. 3. A state-transition diagram – M/M(g)/1/k/S queue

Applying these coefficients to the general solution of the birth and death process and using traffic intensity $a = \lambda/\mu$ we obtain the steady state probabilities

$$P''_j = \frac{a^j \Big/ (j!)^{1-p}}{\sum_{i=0}^{k+1} a^i \big/ (i!)^{1-p}} \qquad j = 0, 1, 2, \ldots, k+1. \tag{12}$$

The offered traffic is calculated by means of the average arrival rate and the mean holding time

$$A = \lambda \bar{\tau} = a \sum_{j=1}^{k+2} \frac{1}{j^{1-p}} P''_{j-1}. \tag{13}$$

The carried traffic is equivalent to the probability that the system is busy:

$$A_o = 1 - P''_0 = A(1 - P''_{k+1}). \tag{14}$$

4. Performance measures.

Generalized full accessibility loss system.

The time congestion B_t describes the fraction of time that all n servers are busy:

$$B_t = P'_n. \tag{15}$$

The call congestion B_c is the fraction of all call attempts which observe all servers busy and could be obtained as the ratio of lost traffic (difference between offered and carried traffic) to the offered traffic.

$$B_c = \frac{A - A_o}{A}. \tag{16}$$

The traffic congestion B_a is the fraction of the traffic that is not carried, and could be obtained as the ratio of the difference between the intended and carried traffic to the intended traffic

$$B_a = \frac{A_i - A_o}{A_i}. \tag{17}$$

Generalized single server delay system.

Blocking probability. The time congestion B describes the fraction of time that all waiting rooms are busy

$$B = P''_{k+1}. \tag{18}$$

Mean number of calls. The mean number of calls present in the system in steady state by definition is

$$L = \sum_{j=1}^{k+1} j\, P''_j. \tag{19}$$

Mean system time. From the Little's formula, we have the mean system time

$$T = L/\lambda. \tag{20}$$

5. Calculation of the state probability. The traffic intensity a is not equal to the intended traffic in a case of a generalized Erlang process when the service time is state dependent because we calculate the power of the Erlang unsymmetrical distribution. That is why we have to calculate the intended traffic A_i and the peakedness z_i when defining the traffic intensity a and peakedness factor p.

From the practical point of view we first define the intended traffic A_i and the peakedness z_i and after that calculate the traffic intensity a and peakedness factor p.

A fundamental question about the system defined by equations (3), (5) and (6) is whether there exist solutions a, p for an arbitrary A_i, z_i. Although there apparently is no formal proof, this seems to be the case and the solution appears to be unique. We can find solutions of the above system with the iterating method of consecutive replacements.

6. Numerical results. In this section we give numerical results obtained by a Pascal program on a personal computer. The described methods were tested on a computer over a wide range of arguments.

Figure 4 shows the generalized Erlang distribution where the intended traffic is A_i = *15 erl*, the number of the sources is $S = 200$ and the peakedness z_i varies from *0.6 to 1.4*. It will be seen that when the peakedness z_i increases the probability distribution becomes broad about the mean.

Figure 5 presents the time congestion in a full availability loss system with 20 servers, 200 sources and different peakedness z_i as a function of the intended traffic A_i. When the intended traffic per server is big $(0.7 - 1$ erl) the influence of the peakedness to the time congestion is negligible.

Figure 6 compares the time, call and traffic congestion probabilities in a full availability loss system with 20 servers, 200 sources and different peakedness of the intended traffic z_i as function of the intended traffic A_i.

Figure 7 shows the stationary probability distribution in a single server queue M/M(g)/1/k with a state dependent mean service time, 40 waiting positions, 0.65 erl traffic intensity and different peakedness factor p. We can see that when the peakedness factor is bigger than one, the probabilities can increase when the number of the calls in the system increases.

Figure 8 illustrates the dependence of the mean service time from the number of calls in the system and different peakedness factor p from 0.7 to 1.15. We can see that when the peakedness factor is smaller or bigger than one, the mean service time decreases or increases respectively when the number of the calls in the system increases.

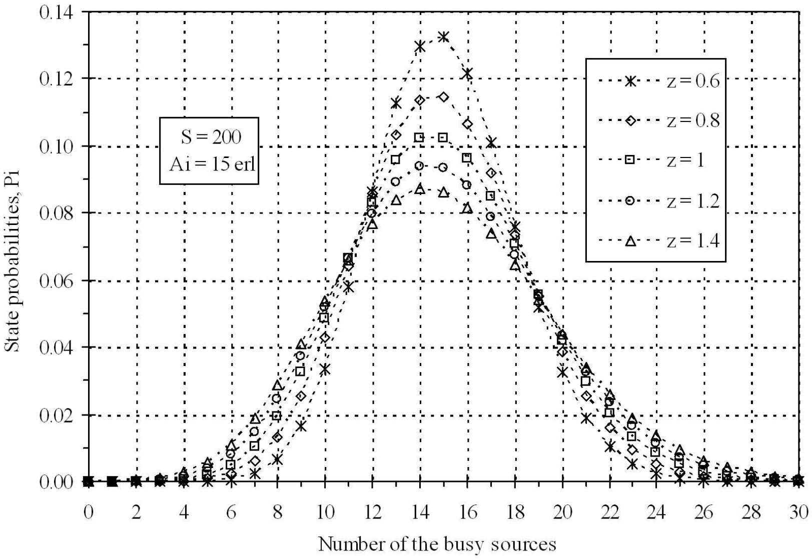


Fig. 4. Generalized Erlang distribution for intended traffic $A_i = 15$ erl, the number of the sources $S = 200$ and different peakednesses z_i

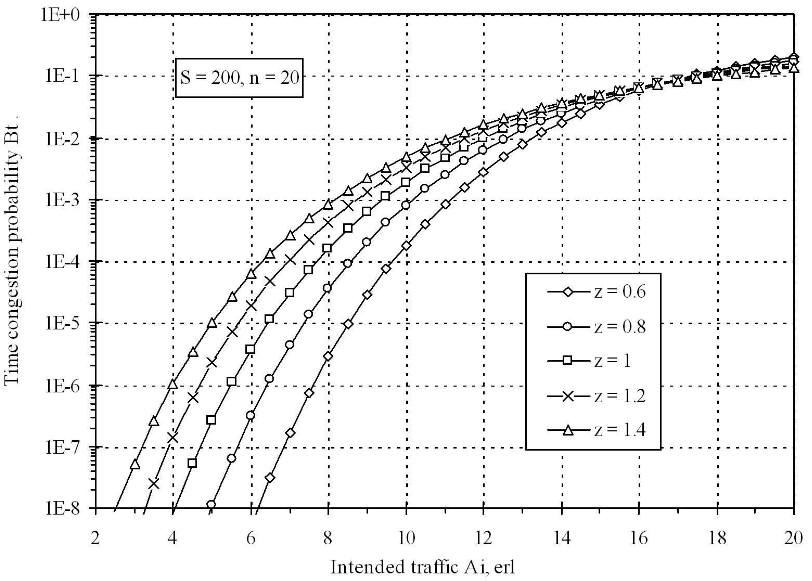


Fig. 5. Time congestion in a full availability loss system with 20 servers, 200 sources and different peakednesses z_i

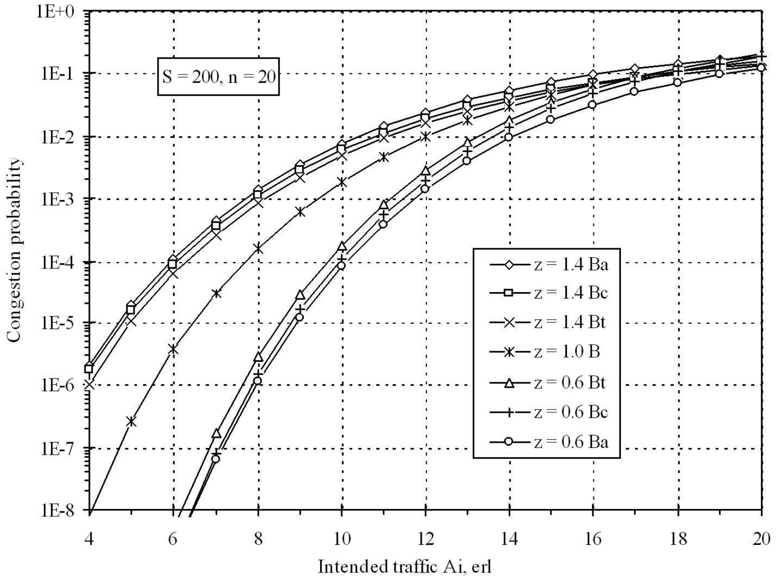


Fig. 6. Time, call and traffic congestion in a full availability loss system with 20 servers, 200 sources and different peakednesses z_i

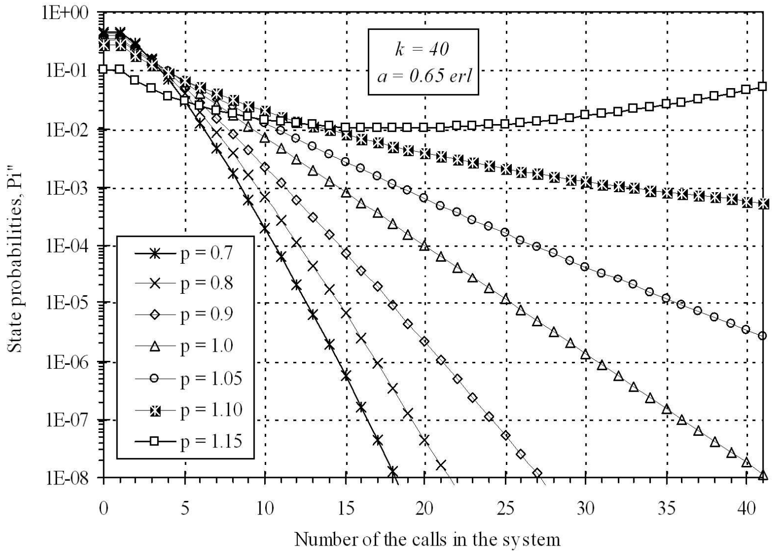


Fig. 7. Stationary probability distribution in a single server queue with state dependent mean service time and different peakedness factors

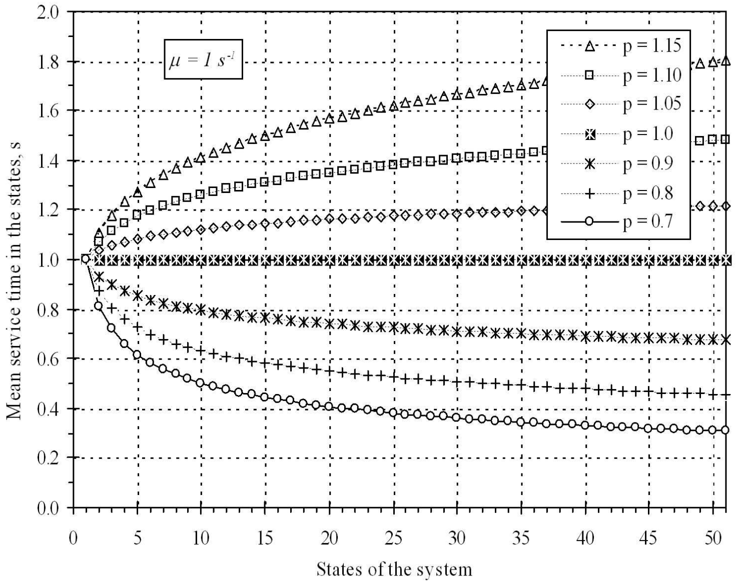


Fig. 8. Dependence of the mean service time from the number of the calls in the system and different peakedness factors

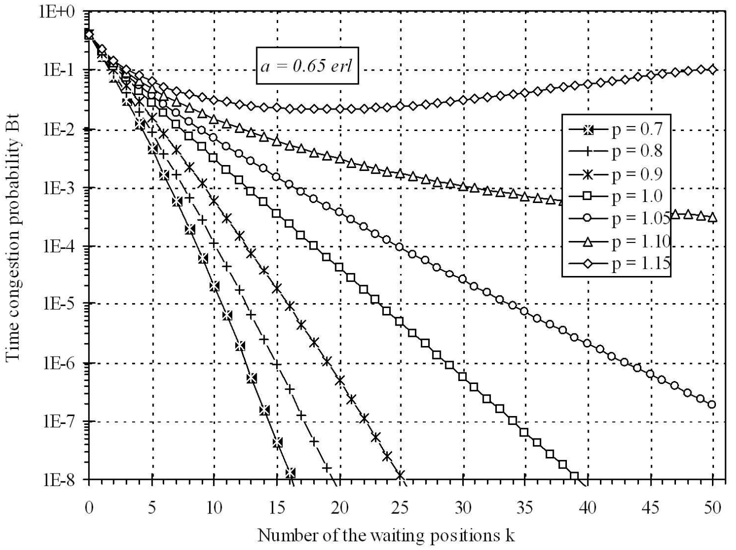
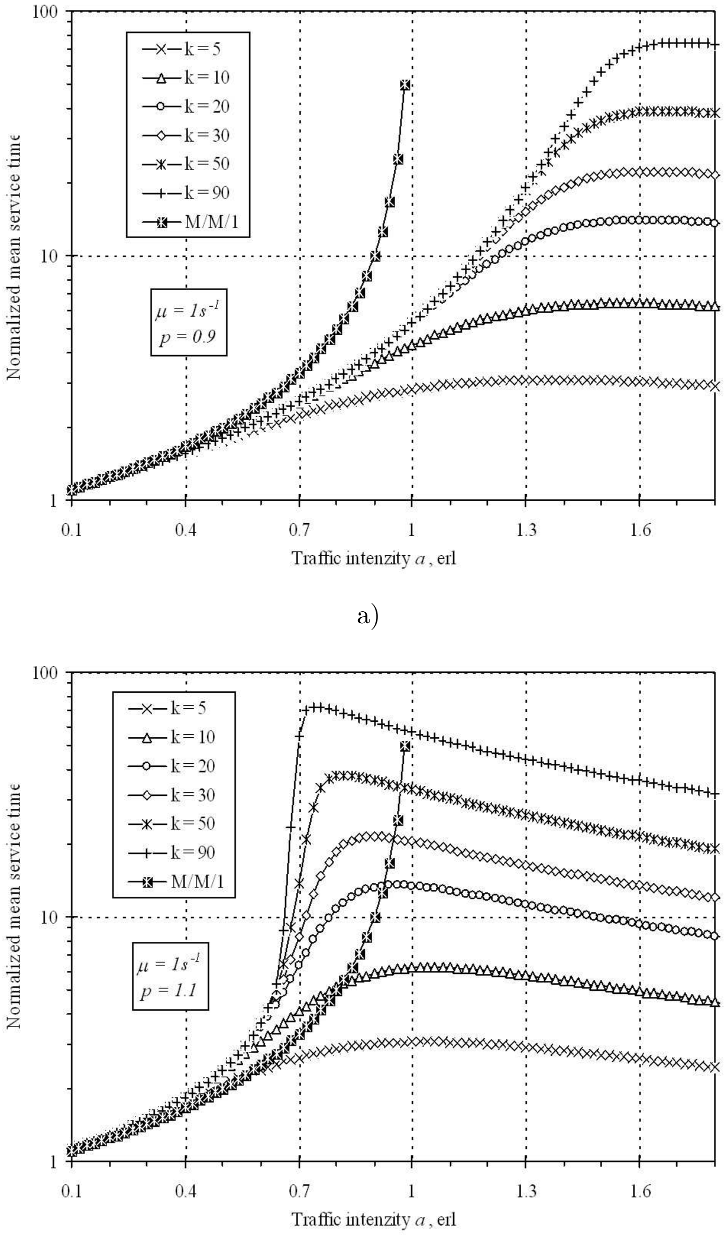


Fig. 9. Time congestion in a single delay system with state dependent mean service time and different peakedness factors



a)

b)

Fig. 10. Normalized mean system time when the mean service time a) decreases and b) increases when the number of the calls in the system increases

Figure 9 shows the time congestion in a single delay system with 0.65 erl traffic intensity and different peakedness factors as function of the buffer size. When the peakedness factor is bigger then 1 the influence of the buffer size on the time congestion is negligible. In some cases the time congestion can increase when the buffer size increases.

Figure 10 (a, b) presents the normalized mean system time ($T' = T/\tau$) as a function of the traffic intensity when the peakedness factor is 0.9 and 1.1 respectively and for different waiting rooms.

It is shown that the influence of the peakedness over the performance measures is significant.

8. Conclusion. In this paper a generalized Erlang distribution as a result of state dependent mean service time is introduced and evaluated. A basic model for a loss system M/M(g)/n/0/S and delay queue M/M(g)/1/k is examined in detail.

The proposed method provides a unified framework to model a peaked, regular and smooth behaviour of the teletraffic systems. Numerical results and subsequent experience have shown that this method is accurate and useful in analysis of queuing systems.

The classic teletraffic system – the full accessibility loss system – is independent of the service time distribution. In this paper it is shown that the influence of the state dependent service rate over the main parameters of the full availability loss system is significant. The main parameters of this system – state probabilities and call, time and traffic congestion – are defined and presented graphically.

The single server delay system with state dependent service rate can be used as a means for controlling and smoothing the data flow into telecommunications networks. This system can be used to explain the behaviour of real traffic regulator as “leaky bucket” and “congestion window”.

The importance of the teletraffic systems in a case of state dependent mean service time comes from its ability to describe behaviour found in up-to-day networks. This is the case in a general teletraffic system, which is an important feature in designing telecommunications networks.

In conclusion, we believe that the presented generalized Erlang distribution and queuing system will be useful in practice. As part of future work, we plan to analyse a regulator in the network.

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